Patterns Free to Point in Any Direction

Slices, Traces, Transits and Two Cuts of One Object

Fedge No

fedge-no@hotmail.com

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Abstract. A direction is something a pattern vibrates in. A particle pattern of depth d vibrates in d bound directions, and those directions can point in any direction of space, including out of the three we observe. A photon is the two-direction case, which is why light travels in every direction of our three. This paper works out what an observer confined to three directions sees of such a pattern. The core of a depth-d pattern meets our three in a set of dimension $3-d$, and in space and time $4-d$: depth two leaves a line, depth three a lasting point, depth four a single instant. The average squared share of an observed direction lying in the pattern is d/D. A tilted free direction moves the trace to distance $h\tan\varphi$, through infinity and back. A bent free line crosses our three in pairs that are born together and separate as $\sqrt{t}$. A field packet moving along a hidden direction appears in our three and vanishes again while its total energy stays fixed, with the slice energy balanced exactly by the flux through the hidden direction. One vibration plane cut at two places gives the correlation $-\cos\theta$ and the CHSH value $2\sqrt{2}$. Each count of directions m carries its own sphere S^{m-1}, and the right-angle field counts are $1,\ \pi/2,\ 2,\ 3\pi/4,\ 8/3$ for $m = 2$ to 6.

1. Setting

Space has infinitely many ordinary directions; matter patterns occupy up to ten of them, so we work in $\mathbb{R}^D$ with $D = 10$. A pattern of depth d has a bound d-plane $B \subset \mathbb{R}^D$, which we represent by a $D \times d$ matrix with orthonormal columns, and a centre p. It is localized within B and extended along the complement $B^\perp$. The projector onto B is $P_B = BB^\mathsf{T}$. The observer's three directions span $O = \mathrm{span}(e_1, e_2, e_3)$. The depths used here are those of the mass rule [1]: photon 2, down quark 3, up quark 4, electron 6.

The plane B is any d-plane. Its orientation relative to O is free and can change with time.

2. The trace of the core in our three

The core of the pattern is the set where its bound coordinates vanish, $\{y : P_B(y-p) = 0\} = p + B^\perp$. Its trace in our three is

$$T = \{x \in O :\ P_B(x-p) = 0\} \quad\Longleftrightarrow\quad (B^\mathsf{T}O)\,x = B^\mathsf{T}p, \tag{1}$$

where O also denotes the $D \times 3$ matrix (e_1, e_2, e_3). This is d linear equations in three unknowns. For a generic orientation and centre the matrix $B^\mathsf{T}O$ has rank $\min(d, 3)$, so T has dimension $3-d$ when $d \le 3$ and is empty when $d > 3$.

If the centre moves, $p(t) = p + vt$ with v having components outside O, the time t is a fourth unknown, and the trace in space and time has dimension $4-d$ for $d \le 4$. Table 1 collects the

cases. A sample of 2000 random cases at each depth $d = 1, \dots, 8$ in $D = 10$ reproduces every entry, with B the orthonormal factor of the QR decomposition of a Gaussian $D \times d$ matrix, p and v standard normal vectors, and a trace counted when the least-squares residual of equation (1) is below 10^{-9}: rank $\min(d, 3)$ in every case, a trace in space for every sample with $d \leq 3$ and for none with $d \geq 4$, and a trace in space and time for every sample with $d \leq 4$ and none with $d \geq 5$.

Depth d	Trace in space	Trace in space and time	Example
1	plane	three-dimensional	
2	line	two-dimensional	photon
3	point, lasting	line (a worldline)	down quark
4	none	point (a single instant)	up quark
≥ 5	none	none	electron (6)

Table 1. Dimension of the core's trace in our three directions, for a generic orientation. In space it is $3 - d$; in space and time it is $4 - d$.

For the photon, $d = 2$, the trace is a line in O whose direction k spans the null space of the 2×3 matrix $B^{\mathsf{T}}O$. Over random orientations the average of kk^{T} is $I/3$, to within 0.004 with 20,000 samples in $D = 3$ and 0.003 in $D = 10$: the line points equally in every direction of our three.

3. How much of a pattern lies in our directions

For a d-plane B with uniformly random orientation in $\mathbb{R}^D$ and any fixed unit vector e, symmetry gives

$$\left\langle |P_B e|^2 \right\rangle = \frac{\operatorname{tr} P_B}{D} = \frac{d}{D}. \tag{2}$$

The d-planes that contain all of O form the Grassmannian $\mathrm{Gr}(d - 3, D - 3)$ of dimension $(d-3)(D-d)$, inside $\mathrm{Gr}(d, D)$ of dimension $d(D-d)$. Containing all three observed directions therefore costs

$$d(D-d) - (d-3)(D-d) = 3(D-d) \tag{3}$$

conditions. Table 2 gives both numbers in ten directions. The electron, with six directions, has the largest share of our three and needs the fewest conditions to hold all of them.

Pattern	Depth d	Mean squared share $d/10$	Conditions to contain O, $3(10-d)$
Photon	2	0.2	—
Down quark	3	0.3	21
Up quark	4	0.4	18
Electron	6	0.6	12

Table 2. Overlap of a freely oriented pattern with our three directions, in $D = 10$. A depth-two pattern cannot contain three directions.

4. A tilted free direction

Take depth three in four directions. Let the free direction be $n = (\sin\varphi, 0, 0, \cos\varphi)$ and the bound directions $e = (\cos\varphi, 0, 0, -\sin\varphi)$, e_2 and e_3, with the centre at $p = (0, 0, 0, h)$. At $\varphi = 0$ the bound directions are e_1, e_2, e_3 and the pattern sits at height h above our three along the fourth direction. The core is the line $p + \tau n$, and it meets O where $h + \tau\cos\varphi = 0$:

$$x_1 = -h\tan\varphi. \tag{4}$$

The trace runs out to infinity as $\varphi \to 90^\circ$ and returns from the other side, and the sense in which the core crosses O, $\operatorname{sign}(\cos\varphi)$, reverses (Table 3, Figure 1). Solving equation (1) numerically at these angles reproduces equation (4) to 1.6×10^{-16}, relative for $|x_1/h| \geq 1$ and absolute below.

φ	0°	30°	60°	80°	89°	91°	100°	120°	150°	180°
x_1/h	0	-0.577	-1.732	-5.671	-57.29	57.29	5.671	1.732	0.577	0
Sense	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$-$	$-$	$-$

Table 3. Position and crossing sense of the trace as the free direction turns.

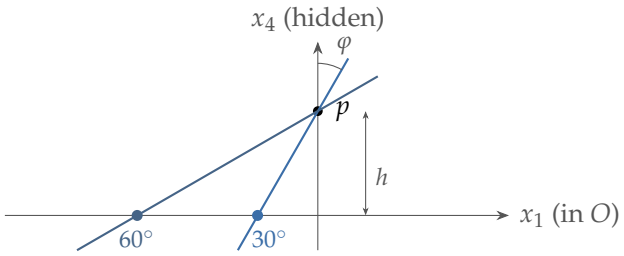


Figure 1. The core of a depth-three pattern seen in the plane of x_1 and the hidden direction x_4. The core is the line through p along the free direction; it meets our three at $x_1 = -h\tan\varphi$. As φ approaches 90° the meeting point runs to infinity.

5. Crossings of a bent free line

Let the free direction bend. Parametrize the free line by $\lambda \in [-1, 1]$ and let $w(\lambda, t)$ be its hidden coordinate, with our three at $w = 0$:

$$w(\lambda, t) = -\lambda + \sum_{k=1}^{8} \left(a_k \cos t + b_k \sin t\right) \sin\Big(\frac{k\pi(\lambda+1)}{2}\Big), \tag{5}$$

with a_k, b_k drawn from a normal distribution of width $1.6/k$. The ends are fixed at $w(\pm 1) = \mp 1$, so the signed number of crossings, $\sum \operatorname{sign}(\partial_\lambda w)$ over the zeros of w, is -1 at all times. When the line folds through $w = 0$ the number of crossings changes by two: crossings are born and lost in pairs of opposite sense. Counting crossings as sign changes of w on 6001 equally spaced values of λ, sixty random lines over a full period $t \in [0, 2\pi]$, at 800 times each, all keep the signed count at -1 and change the count only in steps of two (or four when two folds fall in one time step).

Near a fold, $w = x^2/(2\rho) - vt$, so the pair born at $t = 0$ sits at $x = \pm\sqrt{2\rho v t}$ and separates as

$$\Delta x = 2\sqrt{2\rho v t} \;\propto\; \sqrt{t}. \tag{6}$$

With $\rho = 0.7$, $v = 1.3$ and a cubic term $0.05x^3$ added, the separation of the roots, found by linear interpolation on 1.2×10^6 points of $[-1.2, 1.2]$, over 15 log-spaced times from $t = 10^{-6}$ to 2.6×10^{-4} has log-log slope 0.5000002. In our three, a pattern whose free line bends through us appears as a pair at one point, the two members moving apart as $\sqrt{t}$, and leaves as a pair meeting at one point.

6. A transit through our three

A field moving along a hidden direction passes through our three and leaves. Take one visible direction x and one hidden direction w, our slice being $w = 0$, and a complex field obeying

$$\partial_t^2\phi = \partial_x^2\phi + \partial_w^2\phi - m^2\phi, \qquad m = 1, \tag{7}$$

with energy density and flux

$$u = |\partial_t\phi|^2 + |\partial_x\phi|^2 + |\partial_w\phi|^2 + m^2|\phi|^2, \qquad S_j = -2\,\mathrm{Re}\big(\overline{\partial_t\phi}\,\partial_j\phi\big), \tag{8}$$

which satisfy $\partial_t u + \partial_x S_x + \partial_w S_w = 0$. The energy in our slice is $E_O(t) = \int u(x, 0, t)\,dx$. Integrating the conservation law over x at $w = 0$, the S_x term vanishes and

$$\frac{dE_O}{dt} = -\int \partial_w S_w(x, 0, t)\,dx : \tag{9}$$

energy enters and leaves our three only through the hidden direction.

The packet starts at $w = -15$, moving toward our slice:

$$\phi(x, w, 0) = \exp\Big(-\frac{x^2}{18} - \frac{(w+15)^2}{8}\Big)\,e^{2iw}, \tag{10}$$

evolved exactly in Fourier space with $\omega = \sqrt{m^2 + k_x^2 + k_w^2}$ on a periodic 256×256 grid of side 80. Its group velocity is $2/\sqrt{5} = 0.894$, so it reaches our slice at $t \approx 16.8$. Figure 2 shows the result. The energy in our slice is 8.6×10^{-23} of its peak at the start and 1.4×10^{-5} of it at $t = 32$, with the peak at $t = 16.8$; the total energy is constant to 3.3×10^{-16}; and equation (9) holds to 1.4×10^{-7} of the peak rate $|dE_O/dt| = 15.2$ at $t = 8, 10, \ldots, 18$, with the left side taken by central difference of step 10^{-3} and ∂_w taken spectrally. The pattern comes into existence in our three and goes out of it again, and its total energy is conserved.

The signal recorded at one point of our slice, $\phi(0, 0, t)$ for $t \in [0, 32]$, has duration $\sigma_t = 1.770$ and frequency spread $\sigma_\omega = 0.3128$, the standard deviations of $|\phi|^2$ in t and of its discrete spectrum in ω, from 321 samples at step 0.1, so

$$\sigma_t\,\sigma_\omega = 0.5537 \;\geq\; \tfrac{1}{2}, \tag{11}$$

the time-frequency bound [2]. The carrier frequency is 2.268.

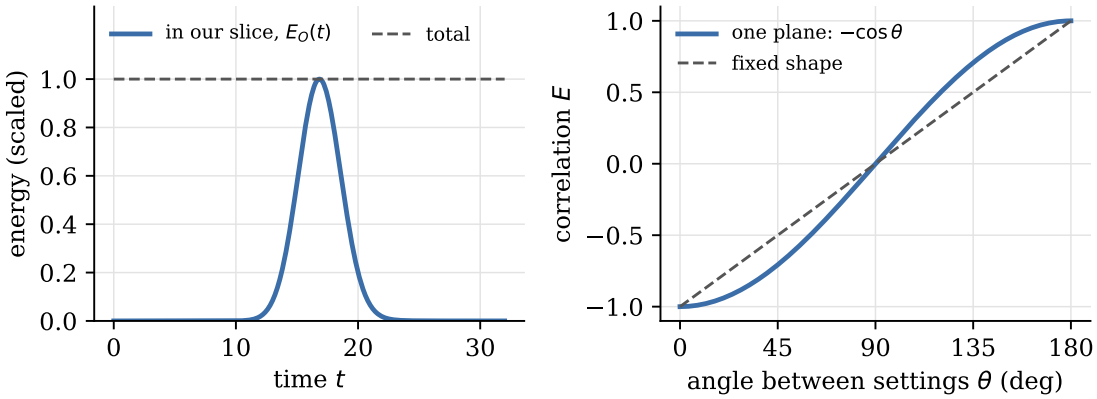


Figure 2. Left: energy in our slice, $E_O(t)$, scaled to its peak, and the total energy, scaled to its initial value, for the packet of equation (10). Right: correlation of two cuts of one object (Section 7). The solid curve is one vibration plane; the dashed line is an object of fixed shape.

7. Two cuts of one object

An object that extends through hidden directions can meet our three in two places at once. Take the object to be one vibration plane in four directions, met by two detectors with settings a and b, unit vectors in our three at angle θ, $\cos\theta = a \cdot b$.

Planes of directions. A setting a selects a real two-plane $\Pi(a) \subset \mathbb{R}^4$: take the eigenvector $c \in \mathbb{C}^2$ of $a \cdot \sigma$ with eigenvalue $+1$, where σ are the Pauli matrices, and let $\Pi(a)$ be spanned by $r(c)$ and $r(ic)$, with $r(u) = (\mathrm{Re}\, u_0, \mathrm{Im}\, u_0, \mathrm{Re}\, u_1, \mathrm{Im}\, u_1)$. The two principal angles between $\Pi(a)$ and $\Pi(b)$ are both $\theta/2$ (checked on 5000 random pairs to 2.8×10^{-15}). The share of a vibration in $\Pi(a)$ that falls in $\Pi(b)$ is half the trace of the product of their projectors,

$$s(a,b) = \tfrac{1}{2}\,\mathrm{tr}\left(P_{\Pi(a)} P_{\Pi(b)}\right) = \cos^2\frac{\theta}{2} = \frac{1 + a \cdot b}{2}. \tag{12}$$

Premises. (i) Both cuts belong to the one plane, with opposite sense. (ii) Contact at the first detector sets the plane to that detector's plane $\Pi(\pm a)$, each with probability $\frac{1}{2}$. (iii) The share of the vibration in a plane sets how often the result belonging to that plane occurs. Then, for results $s_a, s_b \in \{+1, -1\}$,

$$\Pr(s_a, s_b) = \tfrac{1}{2}\, s(-s_a a,\, s_b b) = \tfrac{1}{4}\big(1 - s_a s_b\, a \cdot b\big), \tag{13}$$

which gives

$$E(a,b) = \sum s_a s_b \Pr(s_a, s_b) = -\cos\theta, \qquad \Pr(s_b = +1) = \tfrac{1}{2} \text{ for every } a. \tag{14}$$

At the far detector each result has probability $\frac{1}{2}$ for every near setting, and equation (13) is symmetric in the two detectors, so either may make contact first. With a at 0° and 90° and b at $\pm 45^\circ$ in one plane,

$$\big|E(a_0,b_0) + E(a_0,b_1) + E(a_1,b_0) - E(a_1,b_1)\big| = 2\sqrt{2} = 2.828427. \tag{15}$$

Computing the shares numerically from the plane construction reproduces equations (14) and (15) to 10^{-15}.

Fixed shape. An object whose orientation λ fixes both results in advance, $s_a = \operatorname{sign}(\lambda \cdot a)$ and $s_b = -\operatorname{sign}(\lambda \cdot b)$ with λ uniform, gives the straight line $E = -1 + 2\theta/\pi$ and CHSH value 2 [3, 4]. Random orientations in 3, 4, 6 and 10 directions give the same line and the value 2 to sampling error 10^{-3}. The value $2\sqrt{2}$ is the one measured in entanglement experiments, and it comes here from one plane met in two places (Figure 2, right).

8. Each count of directions has its own sphere

A pattern whose field acts in m directions has the field $Mx/|x^{\mathsf{T}}Mx|^{m/2}$, the gradient of $|x^{\mathsf{T}}Mx|^{-(m-2)/2}$ up to a constant, where M is the pattern's projector restricted to those m directions. Its flux is counted on the sphere S^{m-1} of those m directions. With one direction tilted, by cosine c, $M = \operatorname{diag}(1, \ldots, 1, c^2)$ in principal axes, and on the unit sphere the outward radial component of the field is $Q^{-(m-2)/2}$ with $Q = \sin^2\vartheta + c^2\cos^2\vartheta$, ϑ being the angle from the tilted axis. The count, normalized so that an untilted pattern gives 1, is

$$g_m(c) = \frac{\displaystyle\int_0^{\pi} \sin^{m-2}\vartheta \left(\sin^2\vartheta + c^2\cos^2\vartheta\right)^{-(m-2)/2} d\vartheta}{\displaystyle\int_0^{\pi} \sin^{m-2}\vartheta \, d\vartheta}. \tag{16}$$

For three and four directions, with $s = \sqrt{1-c^2}$,

$$g_3(c) = \frac{\arcsin s}{s}, \qquad g_4(c) = \frac{2}{1+c}. \tag{17}$$

At a right angle, $c = 0$, the integrand's numerator becomes 1 and

$$g_m(0) = \frac{\pi}{\int_0^{\pi} \sin^{m-2}\vartheta \, d\vartheta} = \frac{\sqrt{\pi}\,\Gamma(m/2)}{\Gamma\big((m-1)/2\big)}. \tag{18}$$

Table 4 lists the values. The factor π appears only for odd counts; in four directions the right-angle count is the integer 2.

Directions m	2	3	4	5	6
Sphere	S^1	S^2	S^3	S^4	S^5
Right-angle count $g_m(0)$	1	$\pi/2$	2	$3\pi/4$	$8/3$

Table 4. Right-angle field counts by number of directions, from equation (18), confirmed by quadrature to 10^{-12}.

The tilted field has nonzero divergence in the observed directions whenever $c \neq 1$, so its count depends on the surface it is counted through. On the surface $x^{\mathsf{T}}Mx = 1$, which follows the field's own shape, the count is

$$\frac{2+c^2}{3c} \quad (m=3), \qquad \frac{3+c^2}{4c} \quad (m=4), \tag{19}$$

1.5 and 1.625 at $c = 0.5$, against 1.2092 and $4/3$ on the round sphere. Surfaces $r = e^{\alpha \cos 2\vartheta}$ in three directions at $c = 0.5$, with ϑ measured from the tilted axis and the flux divided by 4π, give -0.046, 0.540, 1.209, 1.879 and 2.464 for $\alpha = -1.5, -0.8, 0, 0.8, 1.5$. Counted in the pattern's own directions, the count is exactly 1 however the pattern is turned.

9. Summary

Result	Value
Trace of a depth-d core in our three	dimension $3 - d$; in space and time $4 - d$
Mean squared share of an observed direction	d/D; electron 0.6, up 0.4, down 0.3, photon 0.2
Conditions to contain our three	$3(D - d)$; electron 12, up 18, down 21
Trace of a tilted free direction	$x_1 = -h \tan \varphi$, through infinity at 90°
Crossings of a bent free line	signed count fixed, pairs separating as $\sqrt{t}$
Transit along a hidden direction	slice energy balanced by hidden flux; total conserved
Two cuts of one plane	$E = -\cos\theta$, CHSH $2\sqrt{2}$
Right-angle field counts, $m = 2$ to 6	1, $\pi/2$, 2, $3\pi/4$, $8/3$

Table 5. The results.

A pattern whose bound directions point freely is seen in our three only through the trace of its core and the slice of its field. Depth fixes the dimension of that trace. Turning moves it, bending creates and removes it in pairs, and motion along a hidden direction brings a pattern into our three and takes it out again with its energy conserved. One plane met in two places carries the full entanglement correlation.

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