The Generation Law

Each Charged Lepton Is a Quark Plus a Neutrino

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Abstract. A particle is a pattern of identical portions bound in d spatial directions on n levels, with $S(n,d) = \binom{n+d-1}{d}$ portions and mass $\mu_d S(n,d)$. In each of the three generations the charged lepton's level is the sum of a quark's level and a neutrino's level: $13 = 3 + 10$, $35 = 20 + 15$ and $23 = 1 + 22$. Geometrically, the quark and the neutrino cover one edge of the lepton's simplex exactly, as two corners. In the counts, Vandermonde's identity assembles the lepton's whole count from its generation members: the neutrino's count enters at its own depth five, and the quark's count at its own depth, four for the up and charm and three for the down. The paper gives the assignments, the identity with a two-line proof, and the three expansions in full.

1. The patterns

A particle is a pattern of identical portions bound in d ordinary spatial directions and arranged on n levels. The portions sit on the lattice points of a d-dimensional simplex of side $n-1$, and their number is

$$S(n,d) = \binom{n+d-1}{d}. \tag{1}$$

The mass of each particle is its family unit times its count [1, 2],

$$m_{n,d} = \mu_d \, S(n,d). \tag{2}$$

The assignments used here are those of Table 1. The down-type quarks are bound in the three observed directions, the up-type quarks in those and a fourth, the neutrinos in five, the electron and muon in six and the tau in ten. The three neutrino levels label the three mass states.

Particle	Depth d	Level n	Count $S(n,d)$	Shape
down	3	1	1	a point
up	4	3	15	4-simplex, side 2
charm	4	20	8,855	4-simplex, side 19
neutrino 1	5	10	2,002	5-simplex, side 9
neutrino 2	5	15	11,628	5-simplex, side 14
neutrino 3	5	22	65,780	5-simplex, side 21
electron	6	13	18,564	6-simplex, side 12
muon	6	35	3,838,380	6-simplex, side 34
tau	10	23	64,512,240	10-simplex, side 22

Table 1. The patterns that enter the generation law.

The charged leptons share one mass unit, $\mu_6 = \mu_{10} = m_e/18{,}564$ [1]. Their masses then follow from their counts alone (Table 2).

Lepton	Count ratio to the electron	From the count (MeV)	Measured (MeV) [3]
electron	1	0.51099895	0.51099895
muon	$3515/17 = 206.7647$	105.6565	105.6583755
tau	$64{,}512{,}240/18{,}564 = 3475.126$	1775.79	1776.93 ± 0.09

Table 2. Charged-lepton masses from the counts with one shared unit. The electron fixes the unit.

2. The law in the levels

In each generation the charged lepton's level is the sum of its quark's level and its neutrino's level:

$$\begin{aligned} &\text{generation 1:} \quad 13 = 3 + 10 \quad \text{(electron = up + neutrino 1)}, \\ &\text{generation 2:} \quad 35 = 20 + 15 \quad \text{(muon = charm + neutrino 2)}, \\ &\text{generation 3:} \quad 23 = 1 + 22 \quad \text{(tau = down + neutrino 3)}. \end{aligned} \tag{3}$$

The geometric meaning is exact. An edge of the lepton's simplex with n levels carries n lattice points [2]. A pattern with a levels placed as the corner at one end of that edge covers a of its points, and a pattern with b levels placed at the other end covers b. The two corners cover the edge with no gap and no overlap exactly when $n = a + b$. In each generation the quark and the neutrino are those two corners (Figure 1).

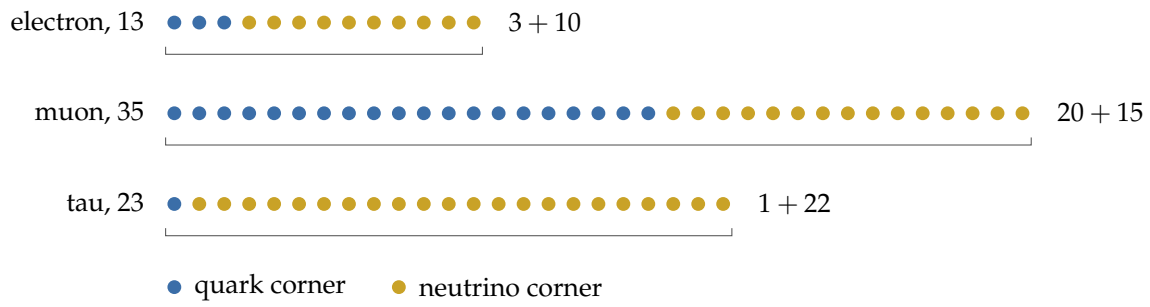


Figure 1. One edge of each charged lepton's simplex, drawn as its lattice points. The quark's corner and the neutrino's corner cover the edge exactly. Electron: up (3) and neutrino 1 (10). Muon: charm (20) and neutrino 2 (15). Tau: down (1) and neutrino 3 (22).

3. The law in the counts

A sum of levels is not a sum of counts. Vandermonde's identity states what a sum of levels means for a count:

$$S(a + b, d) = \sum_{k=0}^{d} S(a, k)\, S(b, d - k). \tag{4}$$

Proof. The counts are the coefficients of a negative binomial series, $\sum_{d \geq 0} S(n, d)\, x^d = (1 - x)^{-n}$. Multiplying the series for a and b gives $(1 - x)^{-a}(1 - x)^{-b} = (1 - x)^{-(a+b)}$, and equating coefficients of x^d gives equation (4). □

Each term divides the parent's d directions between the two corners, k to the first and $d - k$

to the second, and every arrangement of the parent is counted once, in exactly one term. The identity holds for any split of a level. In each generation the neutrino's count appears in the term that gives it five directions, and the quark's count appears in the term that gives it its own depth. Each member appears in its own term.

Generation 1 ($a = 3$, up; $b = 10$, neutrino 1; $d = 6$):

$$S(13,6) = 18{,}564 = 5005 + 3(2002) + 6(715) + 10(220) + 15(55) + 21(10) + 28. \tag{5}$$

The neutrino's count $S(10,5) = 2002$ appears in the $k = 1$ term, times $S(3,1) = 3$. The up quark's count $S(3,4) = 15$ appears in the $k = 4$ term, times $S(10,2) = 55$.

Generation 2 ($a = 20$, charm; $b = 15$, neutrino 2; $d = 6$):

$$\begin{aligned} S(35,6) = 3{,}838{,}380 = 38{,}760 + 20(11{,}628) + 210(3{,}060) + 1540(680) \\ + 8855(120) + 42{,}504(15) + 177{,}100. \end{aligned} \tag{6}$$

The neutrino's count $S(15,5) = 11{,}628$ appears in the $k = 1$ term, times $S(20,1) = 20$. The charm count $S(20,4) = 8{,}855$ appears in the $k = 4$ term, times $S(15,2) = 120$.

Generation 3 ($a = 1$, down; $b = 22$, neutrino 3; $d = 10$). With one level, $S(1,k) = 1$ for every k, and the identity becomes the hockey-stick identity: the tau is a stack of the 22-level patterns at every depth from 0 to 10,

$$\begin{aligned} S(23,10) &= \sum_{j=0}^{10} S(22,j) \\ &= 1 + 22 + 253 + 2{,}024 + 12{,}650 + 65{,}780 + 296{,}010 \\ &\quad + 1{,}184{,}040 + 4{,}292{,}145 + 14{,}307{,}150 + 44{,}352{,}165 = 64{,}512{,}240. \end{aligned} \tag{7}$$

The neutrino's count $S(22,5) = 65{,}780$ is the depth-five layer. The down quark's count $S(1,3) = 1$ is the factor of the term that gives it three directions, $S(1,3)\,S(22,7) = 1{,}184{,}040$.

Gen.	Lepton count	Neutrino term	Quark term
1	$S(13,6) = 18{,}564$	$S(3,1)\,S(10,5) = 3 \times 2002$	$S(3,4)\,S(10,2) = 15 \times 55$
2	$S(35,6) = 3{,}838{,}380$	$S(20,1)\,S(15,5) = 20 \times 11{,}628$	$S(20,4)\,S(15,2) = 8855 \times 120$
3	$S(23,10) = 64{,}512{,}240$	$S(1,5)\,S(22,5) = 1 \times 65{,}780$	$S(1,3)\,S(22,7) = 1 \times 1{,}184{,}040$

Table 3. Where each generation member enters its lepton's count.

4. Summary

The charged lepton of each generation is its quark plus its neutrino. In the levels this is exact addition, equation (3). In the geometry the quark and neutrino are the two corners that cover one edge of the lepton's simplex. In the counts, Vandermonde's identity builds the lepton's whole count from terms in which the neutrino carries five directions and the quark carries its own depth. With one shared unit, the counts of the electron, muon and tau give their masses.

References

[1] F. No, Sector Depth Convergence (2026).

[2] F. No, Particle Decay as the Rearrangement of Simplex Patterns (2026).

[3] Particle Data Group, S. Navas *et al.*, Review of Particle Physics, *Phys. Rev. D* **110**, 030001 (2024), and 2025 update.