Absolute Neutrino Masses from the Depth-Five Simplex Count

The Mass Ratio 2002 : 11628 : 65780 and the Splitting Ratio 0.03035

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Abstract. A particle is a pattern of identical portions bound in d spatial directions on n levels, with $S(n,d) = \binom{n+d-1}{d}$ portions and mass $\mu_d\, S(n,d)$. The three neutrino mass states are depth-five patterns on 10, 15 and 22 levels, so their masses stand in the exact ratio 2002 : 11628 : 65780. The ratio of the two measured mass-squared splittings is then fixed with no free parameter, $\Delta m^2_{21}/\Delta m^2_{31} = 32800595/1080750099 = 0.0303498$, and the ordering is normal. One splitting sets the single scale. Calibrated on $\Delta m^2_{31} = 2.534 \times 10^{-3}\,\mathrm{eV}^2$, the masses are 1.533, 8.903 and 50.362 meV, with sum 60.80 meV, and the solar splitting is $\Delta m^2_{21} = 7.69^{+0.08}_{-0.07} \times 10^{-5}\,\mathrm{eV}^2$. The first JUNO measurement, $(7.50 \pm 0.12) \times 10^{-5}\,\mathrm{eV}^2$, and the global fits agree with this value to within 1.7 to 2.8 percent. The paper gives the counts, the splitting algebra, the absolute spectra for three calibrations, the mass observables Σ, m_β and $m_{\beta\beta}$, and the precision JUNO will reach on the ratio.

1. The mass rule and the neutrino states

A pattern bound in d directions on n levels places its portions on the lattice points of a d-dimensional simplex of side $n-1$. Their number is

$$S(n,d) = \binom{n+d-1}{d} = \sum_{j=0}^{n-1} \binom{j+d-1}{d-1}, \tag{1}$$

the number of nonnegative integer d-tuples with total at most $n-1$; the sum stacks the simplex in layers and is the hockey-stick identity. A family at depth d shares one mass unit μ_d, and each member's mass is

$$m(n,d) = \mu_d\, S(n,d) \tag{2}$$

[1]. The unit μ_d is the family's fundamental frequency in mass units, and the count multiplies it. Depth is the number of ordinary spatial directions the pattern occupies.

The three neutrino mass states are depth-five patterns on

$$(n_1, n_2, n_3) = (10,\ 15,\ 22). \tag{3}$$

These are the levels that complete the generation law [2]: each charged lepton's level is a quark's level plus a neutrino's, $13 = 3 + 10$, $35 = 20 + 15$ and $23 = 1 + 22$. Table 1 gives the counts and the mass ratios.

Mass state	Levels n	Count $S(n,5)$	Ratio to m_1	Mass, μ_5 units
ν_1	10	$\binom{14}{5} = 2{,}002$	1	2,002
ν_2	15	$\binom{19}{5} = 11{,}628$	$5814/1001 = 5.8082$	11,628
ν_3	22	$\binom{26}{5} = 65{,}780$	$230/7 = 32.857$	65,780

Table 1. The three neutrino mass states. Labels $1, 2, 3$ denote mass states. In lowest terms the ratio is $1001 : 5814 : 32890$, and $m_3/m_2 = 5.6570$.

2. The splitting ratio

Oscillation experiments measure differences of squared masses, $\Delta m^2_{ij} = m_i^2 - m_j^2$. With masses $\mu_5 C_i$, where $C_i = S(n_i, 5)$, define the integers

$$A = C_2^2 - C_1^2 = 131{,}202{,}380, \qquad B = C_3^2 - C_1^2 = 4{,}323{,}000{,}396. \tag{4}$$

Then $\Delta m^2_{21} = \mu_5^2 A$ and $\Delta m^2_{31} = \mu_5^2 B$, and the scale cancels from their ratio:

$$\begin{aligned} R \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{31}} &= \frac{A}{B} = \frac{32{,}800{,}595}{1{,}080{,}750{,}099} = 0.030349842, \\ \frac{\Delta m^2_{21}}{\Delta m^2_{32}} &= \frac{A}{B - A} = \frac{32{,}800{,}595}{1{,}047{,}949{,}504} = 0.031299786. \end{aligned} \tag{5}$$

A and B share a factor of 4, and both fractions are in lowest terms. Because the counts increase with n, the ordering is normal, $m_1 < m_2 < m_3$, and the lightest mass is nonzero. Equation (5) holds at every value of μ_5, so it is a pure test of the counts.

3. The absolute spectrum

One measured splitting fixes the scale:

$$\mu_5 = \sqrt{\Delta m^2_{31}/B} \quad \text{or} \quad \mu_5 = \sqrt{\Delta m^2_{21}/A}, \tag{6}$$

and the other splitting is then predicted, $\Delta m^2_{21} = R\,\Delta m^2_{31}$ or $\Delta m^2_{31} = \Delta m^2_{21}/R$. Table 2 gives the spectrum for three calibrations, and Figure 1 shows it on a logarithmic mass scale.

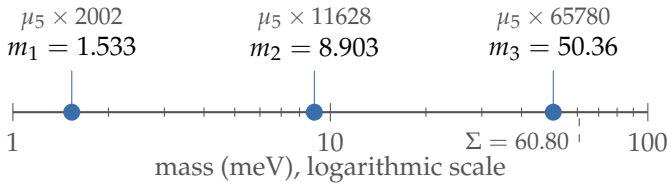


Figure 1. The predicted spectrum for the calibration $\Delta m^2_{31} = 2.534 \times 10^{-3}\,\text{eV}^2$, on a logarithmic scale. Successive masses grow by factors of 5.81 and 5.66. The dashed mark is the sum Σ.

For fixed counts every mass scales as the square root of the calibrating splitting D, so

$$\frac{\sigma_{m_i}}{m_i} = \frac{\sigma_\Sigma}{\Sigma} = \frac{1}{2}\,\frac{\sigma_D}{D}. \tag{7}$$

	Atmospheric calibration		Solar calibration
Quantity	$\Delta m^2_{31} = 2.534 \times 10^{-3}$	$\Delta m^2_{31} = 2.513 \times 10^{-3}$	$\Delta m^2_{21} = 7.50 \times 10^{-5}$
μ_5 (10^{-7} eV)	7.656154	7.624364	7.560661
m_1 (meV)	1.532762	1.526398	1.513644
m_2 (meV)	8.902576	8.865610	8.791537
m_3 (meV)	50.362182	50.153065	49.734029
Σ (meV)	60.797520	60.545072	60.039210
Δm^2_{21} ($10^{-5}\,\mathrm{eV}^2$)	7.690650	7.626915	input
Δm^2_{31} ($10^{-3}\,\mathrm{eV}^2$)	input	input	2.471183

Table 2. Absolute spectra. Splittings are in eV^2. The two atmospheric values are the NuFIT 6.0 normal-ordering results [3]; the solar value is JUNO's first measurement [4]. Extra digits are given so the arithmetic can be reproduced.

For $\Delta m^2_{31} = (2.534^{+0.025}_{-0.023}) \times 10^{-3}\,\mathrm{eV}^2$ this is $+0.49\%/-0.45\%$, and for JUNO's solar value it is 0.80%. The three masses move together. Across all three calibrations the lightest mass is 1.51 to 1.53 meV and the sum is 60.0 to 60.8 meV. Read the other way, JUNO's solar splitting predicts the atmospheric splitting $\Delta m^2_{31} = (2.471 \pm 0.040) \times 10^{-3}\,\mathrm{eV}^2$.

4. Comparison with the measured splittings

Table 3 lists the measurements used.

Quantity	Value	Source
Δm^2_{31}, normal ordering	$(2.534^{+0.025}_{-0.023}) \times 10^{-3}\,\mathrm{eV}^2$	NuFIT 6.0, IC19 without SK-atm [3]
Δm^2_{31}, normal ordering	$(2.513^{+0.021}_{-0.019}) \times 10^{-3}\,\mathrm{eV}^2$	NuFIT 6.0, IC24 with SK-atm [3]
Δm^2_{21}	$(7.49 \pm 0.19) \times 10^{-5}\,\mathrm{eV}^2$	NuFIT 6.0 [3]
Δm^2_{21}	$(7.50 \pm 0.12) \times 10^{-5}\,\mathrm{eV}^2$	JUNO, 59.1 days [4]
Δm^2_{21}	$(7.48 \pm 0.10) \times 10^{-5}\,\mathrm{eV}^2$	Global fit after JUNO [5]
$\sin^2\theta_{12}$	0.3092 ± 0.0087	JUNO [4]
$\sin^2\theta_{13}$	0.02195	NuFIT 6.0, without SK-atm [3]

Table 3. Measured inputs.

The predicted solar splitting is $R\,\Delta m^2_{31}$. Its uncertainty from the atmospheric input is $R\,\sigma_{31}$, which is $+0.076/-0.070$ in units of $10^{-5}\,\mathrm{eV}^2$ for the first atmospheric value and $+0.064/-0.058$ for the second. To compare it with a measurement $\Delta m^2_{21} \pm \sigma_{21}$, the paper uses

$$z = \frac{R\,\Delta m^2_{31} - \Delta m^2_{21}}{\sqrt{\sigma^2_{21} + R^2\sigma^2_{31}}}, \tag{8}$$

with σ_{31} symmetrized to 0.024×10^{-3} and $0.020 \times 10^{-3}\,\mathrm{eV}^2$ for the two atmospheric values and correlations neglected. Table 4 gives the result, and Figure 2 shows it.

The prediction agrees with every current determination to within 1.7 to 2.8 percent, at z between 0.7 and 1.7. Expressed as the scale, the two splittings give values of μ_5 that agree to within 0.8 to 1.4 percent; this is the same comparison through a square root.

Δm^2_{31} input ($10^{-3}\,\mathrm{eV}^2$)	Predicted Δm^2_{21} ($10^{-5}\,\mathrm{eV}^2$)	vs NuFIT 6.0 7.49 ± 0.19	vs JUNO 7.50 ± 0.12	vs global after JUNO 7.48 ± 0.10
2.534	$7.691^{+0.076}_{-0.070}$	$+2.68\%$, $z = 0.99$	$+2.54\%$, $z = 1.36$	$+2.82\%$, $z = 1.70$
2.513	$7.627^{+0.064}_{-0.058}$	$+1.83\%$, $z = 0.69$	$+1.69\%$, $z = 0.94$	$+1.96\%$, $z = 1.26$

Table 4. The predicted solar splitting against three determinations. Percentages are prediction over measurement minus one; z is equation (8).

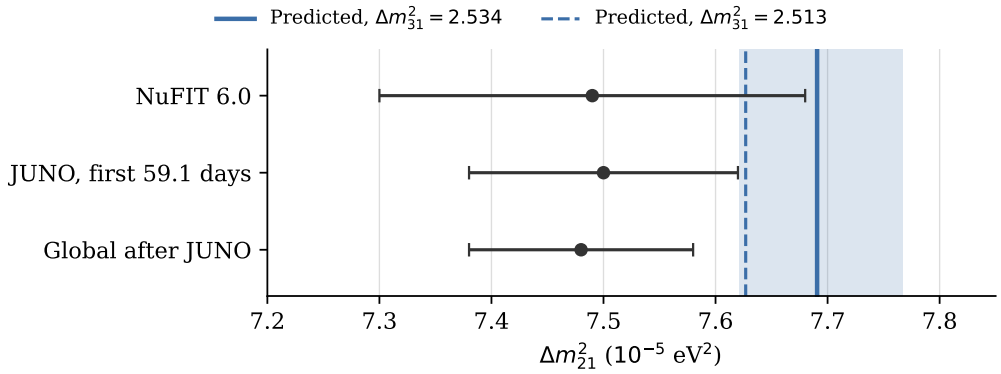


Figure 2. Measured solar splittings (points, one standard deviation) and the prediction $R\,\Delta m^2_{31}$. The solid line and band use $\Delta m^2_{31} = 2.534 \times 10^{-3}\,\mathrm{eV}^2$ with its atmospheric uncertainty; the dashed line uses $2.513 \times 10^{-3}\,\mathrm{eV}^2$.

5. Mass observables

The sum. In normal ordering the sum has the floor $\sqrt{\Delta m^2_{21}} + \sqrt{\Delta m^2_{31}}$, reached as $m_1 \to 0$. With $\Delta m^2_{21} = 7.50 \times 10^{-5}\,\mathrm{eV}^2$ the floor is 59.00 meV for $\Delta m^2_{31} = 2.534 \times 10^{-3}$ and 58.79 meV for 2.513×10^{-3}. The predicted sums lie 1.80 and 1.75 meV above these floors; the solar calibration gives 1.67 meV above its own floor. The excess is the nonzero lightest mass, $m_1 \simeq 1.5$ meV, together with its effect on m_2 and m_3. The predicted sum, 60.0 to 60.8 meV, lies below the DESI DR2 bound $\Sigma < 64.2$ meV (95%, ΛCDM, DESI with CMB) [6].

Beta decay. With the standard electron-row weights

$$\begin{aligned} |U_{e1}|^2 &= \cos^2\theta_{12}\cos^2\theta_{13} = 0.67564, \\ |U_{e2}|^2 &= \sin^2\theta_{12}\cos^2\theta_{13} = 0.30241, \\ |U_{e3}|^2 &= \sin^2\theta_{13} = 0.02195, \end{aligned} \tag{9}$$

from Table 3, the effective beta-decay mass is

$$m_\beta = \Big(\sum_i |U_{ei}|^2 m_i^2\Big)^{1/2} = 9.01\,\mathrm{meV}, \tag{10}$$

and 8.98 meV for the second atmospheric value with the same mixing weights. The KATRIN limit is $m_\nu < 0.45$ eV (90%) [7].

Neutrinoless double beta decay. If the states are Majorana with unknown phases, $m_{\beta\beta} = \left|\sum_i |U_{ei}|^2 m_i e^{i\alpha_i}\right|$ ranges between

$$m_{\beta\beta}^{\max} = \sum_i |U_{ei}|^2 m_i, \qquad m_{\beta\beta}^{\min} = \max\Big(0,\ 2\max_i |U_{ei}|^2 m_i - \sum_i |U_{ei}|^2 m_i\Big), \tag{11}$$

which gives $0.55\,\text{meV} \le m_{\beta\beta} \le 4.83\,\text{meV}$ for the first calibration and 0.55 to 4.81 meV for the second, with the same weights. For the first calibration the three terms $|U_{ei}|^2 m_i$ are 1.036, 2.692 and 1.105 meV. The largest, $|U_{e2}|^2 m_2$, exceeds the other two together by 0.551 meV, so $m_{\beta\beta}$ stays above zero for every choice of phases.

6. Depth five

The same three levels at other depths give the ratios in Table 5. The measured ratio, over the three solar and two atmospheric values of Table 3, lies between 0.0295 and 0.0298. Depth five is the depth that reproduces it.

Depth d	$\big[S(15,d)^2 - S(10,d)^2\big] \,/\, \big[S(22,d)^2 - S(10,d)^2\big]$
4	0.055497
5	0.030350
6	0.016865

Table 5. The splitting ratio for the levels $(10, 15, 22)$ at neighbouring depths.

7. The measurement ahead

JUNO measures Δm^2_{21} and the large splitting in one detector and projects better than 0.5% precision on each within six years of data [8]. Two independent 0.5% measurements fix their ratio to $\sqrt{2} \times 0.5\% = 0.71\%$, against the predicted $R = 0.0303498$. The same data sharpen the ordering, for which a global analysis including the first JUNO data already prefers normal ordering at $\Delta\chi^2 = 4.6$, and 9.4 with the atmospheric data [9]. The ordering and the lightest mass are both fixed here: normal, with $m_1 = 1.51$ to $1.53\,\text{meV}$.

8. Summary

The depth-five counts on 10, 15 and 22 levels fix the neutrino spectrum up to one scale. The splitting ratio 0.03035 follows with no free parameter and agrees with the first JUNO measurement and the global fits to within 1.7 to 2.8 percent. The spectrum is normally ordered, the lightest mass is 1.5 meV, and the sum is 60.0 to 60.8 meV.

Quantity	Prediction
Mass ratio $m_1 : m_2 : m_3$	2002 : 11628 : 65780 (exact)
$\Delta m^2_{21}/\Delta m^2_{31}$	$32800595/1080750099 = 0.0303498$
$\Delta m^2_{21}/\Delta m^2_{32}$	$32800595/1047949504 = 0.0312998$
Ordering	normal
Masses, $\Delta m^2_{31} = 2.534 \times 10^{-3}$	1.533, 8.903, 50.362 meV
Sum Σ	60.0 to 60.8 meV
Δm^2_{21} from $\Delta m^2_{31} = 2.534 \times 10^{-3}$	$7.69^{+0.08}_{-0.07} \times 10^{-5}\,\mathrm{eV}^2$
Δm^2_{31} from JUNO's Δm^2_{21}	$(2.471 \pm 0.040) \times 10^{-3}\,\mathrm{eV}^2$
m_β	9.0 meV
$m_{\beta\beta}$ (Majorana)	0.55 to 4.83 meV

Table 6. The predictions. One scale is calibrated; everything else follows from the counts 2002, 11628 and 65780.

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