Particle Decay as the Rearrangement of Simplex Patterns

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Abstract

In IDWT a particle is a pattern of identical portions, bound in d spatial directions and arranged on n levels. The portions occupy the lattice points of a d-dimensional simplex of side $n - 1$, and their number is the count $C = S(n, d) = C(n + d - 1, d)$. The same shapes have already given the quark mixing matrix. This paper shows that particle decay is a rearrangement of these shapes, through two exact geometric operations. In the first, the parent splits along one of its edges into two corner patterns that cover that edge exactly, so the parent's level is the sum of the two daughters' levels. This is the muon's decay, the tau's electron and hadronic channels, the non-leptonic strange decay and the charmless semileptonic bottom decay to an electron. In the second, a pattern gains or loses the fourth direction and passes between a four-dimensional quark and a three-dimensional piece of it. This is the quark side of beta decay and of the dominant decays of charm, bottom and top. The decays that cross between the first two generations are suppressed by the Cabibbo corner overlap. Section 6 lists the observed decays that these two operations do not yet cover.

1. The patterns

Particle	Depth d	Level n	Count C	Shape
down	3	1	1	a point
strange	3	4	20	tetrahedron, side 3
bottom	3	16	816	tetrahedron, side 15
up	4	3	15	4-simplex, side 2
charm	4	20	8,855	4-simplex, side 19
top	4	72	1,215,450	4-simplex, side 71
neutrino 1	5	10	2,002	5-simplex, side 9
neutrino 2	5	15	11,628	5-simplex, side 14
neutrino 3	5	22	65,780	5-simplex, side 21
electron	6	13	18,564	6-simplex, side 12
muon	6	35	3,838,380	6-simplex, side 34
tau	10	23	64,512,240	10-simplex, side 22

The mass of each particle is its family unit times its count. The down-type quarks are bound in the three observed directions; the up-type quarks in those and the fourth; the neutrinos in five directions; the electron and muon in six; the tau in ten. The bottom quark's mass sits between 16 and 17 levels (the flavor paper); this paper uses its 16-level pattern.

The three neutrino patterns are the three mass states. A neutrino produced or absorbed with a charged lepton is a flavor state, a mixture of the three. In each decay below, the neutrino named is the mass state whose pattern the split uses.

2. The geometry used

All statements in this section are exact.

Faces. Every face of the d-simplex with n levels that spans j of its directions is the j-simplex with n levels, with S(n, j) points. A pattern of depth j fits inside a pattern of depth d when $j \le d$ and its level is at most n.

Corners. The corner of side m at any vertex is the simplex with $m + 1$ levels. A lighter pattern of the same depth is a corner of a heavier one.

Sides. Breaking one side off the simplex with n levels leaves the simplex with $n - 1$ levels. This is Pascal's rule:

$S(n, d) = S(n, d - 1) + S(n - 1, d)$.

Layers. A pattern with n levels in d directions is a stack of patterns in $d - 1$ directions with 1, 2, ..., n levels:

$S(n, d) = S(1, d - 1) + S(2, d - 1) + \ldots + S(n, d - 1)$.

Interiors. Removing all $d + 1$ sides at once leaves the simplex with $n - d - 1$ levels.

3. Edge splits

3.1 The rule

An edge of the parent with n levels carries n lattice points. Place a daughter with a levels as the corner at one end of the edge and a daughter with b levels as the corner at the other end. The two corners cover a and b points of the edge. They cover it exactly, with no gap and no overlap, when

$n = a + b$.

The counting statement of the same split is Vandermonde's identity,

$S(a + b, d) = \text{sum over } k \text{ of } S(a, k) \cdot S(b, d - k)$,

whose terms are the ways of dividing the parent's d directions between the two daughters: k directions to one, $d - k$ to the other. When one daughter has a single level ($b = 1$), every $S(1, d - k)$ is 1 and the identity becomes the Hockey-Stick Identity, $S(n + 1, d) = \text{sum over } k \text{ of } S(n, k)$: the parent is a stack of the patterns with n levels at every smaller depth.

3.2 The observed edge splits

In each decay below there are three products, two particles and one antiparticle, and the two particles are the corners that cover the parent's edge.

Decay	Levels	Measured fraction
muon → electron + antineutrino + neutrino	35 = 13 + 22 (neutrino 3)	about 100%
tau → electron + antineutrino + neutrino	23 = 13 + 10 (neutrino 1)	17.85 ± 0.04%
tau → neutrino + hadrons (down + anti-up)	23 = 22 (neutrino 3) + 1	64.78% in all hadronic channels
strange → up + down + anti-up	4 = 3 + 1	for example $K^+ \to \pi^+\pi^0$, 20.67 ± 0.08%
bottom → up + electron + antineutrino	16 = 3 + 13	$(1.88 \pm 0.27) \times 10^{-3}$ per lepton type

Lepton decays. In the muon and tau decays both daughters fit inside the parent, each in the face

of the parent at its own depth:

Parent	Daughter	Face of the parent at the daughter's depth	Daughter
muon	electron	3,838,380 points	18,564
muon	neutrino 3	575,757 points	65,780
tau	electron	376,740 points	18,564
tau	neutrino 3	80,730 points	65,780
tau	down	2,300 points	1

The tau's hadronic split, 23 = 22 + 1, breaks one side off the tau: the tau with one side removed is the 10-simplex with 22 levels, and its depth-5 face is the third neutrino.

Quark decays. In the strange and bottom splits the daughters are deeper than the parent: the up quark is four-dimensional, and in the bottom decay the electron is six-dimensional, while the parent is a tetrahedron. These splits share the parent's edge but also gain directions, the operation of section 4.

4. Gaining and losing the fourth direction

A down-type quark is a tetrahedron. An up-type quark is a four-dimensional simplex, and its pieces of depth three are its layers and its tetrahedral sides. A weak transition between the two kinds is a pattern gaining or losing the fourth direction: a four-dimensional pattern becomes one of its three-dimensional pieces, or a three-dimensional pattern becomes the four-dimensional pattern it is a piece of.

Decay	Geometry
down → up (beta decay, quark side)	down is the first layer of up
charm → strange (dominant charm decay)	strange is the fourth layer of charm
bottom → charm (dominant bottom decay)	bottom is exactly the interior of one tetrahedral side of charm: removing the surface of a 20-level tetrahedron leaves 816 points, the 16-level tetrahedron
top → bottom (dominant top decay)	bottom is the sixteenth layer of top

Each of these is a pattern and one of its own three-dimensional pieces. The layer relation also connects charm to down (its first layer), top to down and top to strange, so the shapes allow those transitions as well. Which of the allowed transitions dominates is set by the mixing amplitudes of the flavor paper, whose largest entries are these diagonal pairs.

5. Suppressed decays

The decays that cross between the first two generations carry the Cabibbo amplitude, which is the corner overlap of down in strange:

sqrt(C_down / C_strange) = sqrt(1/20) = 0.2236.

The measured |V_us| is 0.2243 ± 0.0008.

- **strange → up.** Strange is not a layer of up, which has three levels. The transition reaches up through down, the corner point of strange, which is the first layer of up. Its amplitude is the corner overlap, √(1/20).
- **charm → down.** The same corner relates down and strange inside charm, and the measured |V_cd|, 0.2247 ± 0.0022, is the same 0.22.

The amplitude for a pattern to be found in the configuration of a smaller pattern it contains is the square root of the fraction of its portions that the smaller one holds; this is the rule that gives all three mixing angles in the flavor paper. The 2–3 and 1–3 crossings carry the corner overlaps of up in charm and up in top, √(15/8,855) and √(15/1,215,450).

6. Decays the two operations do not yet cover

The edge split and the gain or loss of a direction account for the decays above. The following observed decays are not yet accounted for:

- **The lepton pair in beta decay and in the other semileptonic quark decays.** Beta decay (down → up + electron + antineutrino) has 1 < 3 + 13. Strange → up + electron + antineutrino has 4 < 3 + 13. The quark side of these decays is a gain of the fourth direction (section 4); the creation of the lepton pair is not described by either operation. The same holds for the dominant semileptonic bottom decay, bottom → charm + lepton + antineutrino (B mesons: 10.82 ± 0.15% per lepton type, almost all of it through charm), and for the semileptonic charm decays.
- **The muon and tau channels of the charmless bottom decay.** The split 16 = 3 + 13 works for the electron. The muon (35 levels) and tau (23 levels) are larger than the bottom's 16, while the measured rate is the same for the electron and the muon.
- **Leptonic meson decays.** $K^+ \to \mu^+\nu$ (63.56 ± 0.11%) and the pion's decay to a muon and a neutrino.
- **The tau's muon channel.** The tau decays to a muon, an antineutrino and a neutrino in 17.37 ± 0.04% of decays. The muon does not fit inside the tau: it has 3,838,380 portions, and the largest six-direction face of the tau holds 376,740. Its level, 35, is larger than the tau's 23, so the decay is not an edge split.
- **Strange tau decays.** The tau's hadronic channels with a strange quark.

7. Summary

Two geometric operations on the simplex patterns account for a set of the observed weak decays:

1. **Edge splits.** The parent splits along an edge into two corner patterns that cover it exactly, and the parent's level is the sum of the daughters'. These are the muon decay, the tau's electron and hadronic channels, the non-leptonic strange decay and the charmless semileptonic bottom decay to an electron.
2. **Gaining or losing the fourth direction.** A pattern passes between a four-dimensional quark and a three-dimensional layer or side interior of it. These are the quark side of beta decay and the dominant decays of charm, bottom and top.

The decays that cross between the first two generations carry the Cabibbo corner overlap, √(1/20). The same corners and layers give the quark mixing matrix.

8. Open items

- The creation of the lepton pair in semileptonic decays, the muon and tau channels of the charmless bottom decay, the leptonic meson decays and the tau's muon channel (section 6).
- The rule that turns a split into a rate. The largest Vandermonde term of the tau's hadronic split holds 11/16 = 68.75% of the tau's portions, against the measured 64.78%, and the term in which the electron keeps its own six directions in the tau's electron split holds 20.6%, against 17.85%. The same reading gives 0.48% for the muon's own decay, which is 100%, so a term's share of the portions is not the decay fraction.