Planck’s Constant as the Field Norm, and the Family Units

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Abstract

In IDWT every particle is a stationary solution of one field law, and every property worked out for a particle so far is proportional to one number, the solution’s total size or norm N: its spin is N/2, its charge is a fixed multiple of N, and its energy is its frequency times N, up to a small binding correction. Physics states the same three facts with Planck’s constant in place of N. If every particle carries the same norm, Planck’s constant is that norm. This paper shows how the norm depends on the solution’s frequency at each depth: near threshold N = c_d (m₀ − ω)^(9 − d/2), with one constant c_d per depth. A universal norm therefore fixes one frequency for each depth. The paper then gives the family units, the mass per portion of each particle family, with their frequencies and wavelengths. The lepton unit is the hartree to 1.16%: the electron’s count times the square of the fine-structure constant is 0.989. The boson unit is a million times the lepton unit to 0.26%.

1. The norm and Planck’s constant

A particle of depth d is a localized solution of the field law in d bound directions. Its two radial components F and G oscillate at a frequency ω just below the threshold m₀, and the field’s self-interaction grows with the local density as g (F² + G²)^(1/9). The norm is the total size of the solution,

N = ∫ (F² + G²) d^d x.

Three properties are proportional to it:

Property	In the field law	In physics
spin	N/2	ħ/2
charge	a fixed multiple of N	a fixed unit
energy	ω N, less a small binding share	ħ ω

The measured exactness of these units requires every particle to carry the same norm. The proton and electron charges cancel to about one part in 10^{21}, so their norms are equal to that precision. With one norm for every particle, ħ is that norm, and the count law m = unit × C becomes a mass law: the norm stays fixed while the frequency rises with the count.

2. How the norm depends on the frequency

Write s = m₀ − ω for the gap below threshold. The self-interaction g ρ^(1/9) balances the gap when the density ρ is of order s^9, and the solution’s size is then of order s^(−1/2). The norm, density times volume, scales as

N = c_d s^(9 − d/2).

The constant c_d is a property of the limiting solution at each depth (m₀ = g = 1). Integrating the field law at four gaps, from 0.01 to 0.0005, gives:

Depth d	Exponent 9 − d/2	Fitted exponent	c_d
2	8.0	8.0000	94.02
3	7.5	7.4998	870.4
4	7.0	6.9995	7,794
5	6.5	6.4991	67,330
6	6.0	5.9984	559,700
10	4.0	3.9922	1.696×10^9

The exponents are exact in the limit and the fitted ones approach them as the gap closes. An independent integration reproduces c_3 and c_6 to the printed digits.

Three consequences:

- **One frequency per depth.** With N fixed at the universal value ħ, each depth has one gap, s_d = (ħ/c_d)^(1/(9 − d/2)), and so one frequency.
- **The coupling.** At fixed gap the norm scales as g^(−9), so the universal norm and the coupling fix each other.
- **Depth eighteen.** Below eighteen bound directions the norm vanishes as the gap closes; at eighteen the scaling is neutral, and above it the norm grows without bound. Every particle depth in IDWT, from 2 to 10, is on the vanishing side.

3. The family units

The mass of each particle is its family unit times its count, C = S(n, d). The unit of each family, the mass of one portion:

Family	Members (count)	Unit	Frequency	Wavelength
charged leptons	electron (18,564), muon (3,838,380), tau (64,512,240)	27.532 eV	6.657×10^{15} Hz	45.03 nm
down-type quarks	down (1), strange (20)	4.67 MeV	1.130×10^{21} Hz	0.265 pm
up-type quarks	up (15), charm (8,855), top (1,215,450)	0.1433 MeV	3.464×10^{19} Hz	8.66 pm
bosons	W (2,926), Z (3,321), Higgs (4,560)	27.455 MeV	6.639×10^{21} Hz	45.16 fm

Within each family the units of the members agree: the leptons to 0.06%, the bosons to 0.09%, the up-type quarks to 1.4% and the down-type quarks to 1.2%. These agreements are the count law itself: the counts are the integers that the mass ratios pick out. The muon's count over the electron's is 206.7647, against the measured mass ratio 206.7683, and strange over down is 20, against 19.8.

Two results connect a family to something outside it.

The lepton unit is the hartree. The hartree, the energy scale of the atom, is 27.211 eV, with frequency 6.580×10^{15} Hz and wavelength 45.56 nm. The lepton unit is 1.16% above it. The hartree is α^2 times the electron's mass, so the same statement reads

C_electron × α^2 = 18,564 / 137.036^2 = 0.98856.

The electron's count is the inverse square of the fine-structure constant, 18,779, to 1.14%.

The boson unit is a million lepton units. The boson unit, 27.455 MeV, is 0.99739×10^6 times

the lepton unit. Its wavelength, 45.16 fm, is the lepton unit's 45.03 nm scaled down by a million.

The down-type unit is the down quark's own mass, since its count is 1. The up-type unit is the up quark's mass over 15.

4. The coupling each family would need

If each family's unit were the binding energy of a hydrogen-like state of one of its members, the coupling would be √(unit/mass):

Family	Member	Coupling needed	Measured coupling
leptons	electron	0.0073	α = 0.0073
up-type	up	0.258	strong, about 0.40 (with the colour factor)
down-type	down	1.000	strong, about 0.40
bosons	W	0.0185	weak, α_W = 0.0316

The electron's is the fine-structure constant, which is the hartree statement of section 3 again. The other three are not their families' couplings.

5. Summary

Every property of a particle in the field law is proportional to its norm, and a norm shared by every particle plays exactly the role of Planck's constant. Near threshold the norm at each depth is c_d $(m_0 - \omega)^{(9 - d/2)}$, so a universal norm fixes one frequency per depth and ties the coupling to it. The particle masses divide into family units. The lepton unit is the hartree to 1.16%, which is the electron's count matching $1/\alpha^2$ to 1.14%, and the boson unit is a million lepton units to 0.26%.

6. Open items

- The value of the universal norm. The scaling shows how the norm depends on the frequency at each depth; the field law does not yet select one value, and that selection is the same question as what selects the levels n.
- Whether the allowed levels are the frequencies at which the norm takes the universal value.
- The 1.14% between the electron's count and $1/\alpha^2$, and the 0.26% between the boson unit and a million lepton units.