Quark Flavor Mixing from the Simplex Geometry of the Counted Patterns

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Abstract

In IDWT every particle is a pattern of identical portions, bound in d spatial directions and arranged on n levels. The number of portions is the count C = S(n, d) = C(n + d − 1, d), and the particle’s mass is a family unit times that count. The count is the number of lattice points of a d-dimensional simplex of side n − 1. This paper shows that the quark mixing matrix follows from that shape. The lighter quark of each pair sits in a corner of the heavier quark of the same depth. The three mixing angles are the three corner overlaps, the square roots of the counts’ ratios. The CP phase is the angle at which two sides of a regular tetrahedron meet. With five counts fixed by the masses and no fitted constant, the construction gives eight of the nine matrix elements within 2 percent, the Wolfenstein constants λ and A within 0.7 and 0.4 percent, and the unitarity-triangle angles α and γ and the CP strength J at the measured size. The same geometry gives the dominant pairing of the quark generations, and it places the bottom quark exactly inside a side of the charm pattern.

1. The patterns and their shape

A pattern bound in d directions with n levels holds one portion for each way of distributing up to n − 1 units over the d directions:

$$C = S(n, d) = C(n + d - 1, d).$$

These are the lattice points of a d-dimensional simplex of side k = n − 1: points, segments, triangles, tetrahedra and their four-dimensional analogue as d runs from 0 to 4.

The quark assignments, fixed by the mass fit with one family unit for each depth:

Quark	Depth d	Level n	Count C	Shape
down	3	1	1	a single point
strange	3	4	20	tetrahedron of side 3
bottom	3	16, 17 (mass midway)	816, 969	tetrahedron of side 15, 16
up	4	3	15	4-simplex of side 2
charm	4	20	8,855	4-simplex of side 19
top	4	72	1,215,450	4-simplex of side 71

Down-type quarks are tetrahedra: they are bound in the three observed directions. Up-type quarks are four-dimensional simplices: they are bound in the three observed directions and the fourth. A weak transition between an up-type and a down-type quark is a pattern gaining or losing the fourth direction.

2. Exact geometry of the simplex patterns

Four exact statements are used. Each was checked numerically for the quark shapes.

2.1 Sides are levels. Breaking one side off a simplex with n levels leaves the simplex with n − 1 levels:

$$S(n, d) - S(n, d - 1) = S(n - 1, d).$$

A side of the d-simplex is itself a (d − 1)-simplex with the same n.

2.2 Layers. A four-dimensional pattern is a stack of three-dimensional layers, one for each level:

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S(n, 4) = S(1, 3) + S(2, 3) + … + S(n, 3).
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The layers of charm are tetrahedra with 1, 2, …, 20 levels. The first is down and the fourth is strange.

2.3 Corners. The corner of side j of a simplex is the simplex with j + 1 levels. So every lighter pattern of the same depth sits in a corner of every heavier one:

Heavier	Sides broken off	Corner left	Points in the corner
strange	3	down	1 of 20
charm	17	up	15 of 8,855
top	69	up	15 of 1,215,450
top	52	charm	8,855 of 1,215,450

2.4 Interiors. Taking every side off at once leaves the interior, the simplex with n − d − 1 levels. For tetrahedra this steps the level down by 4.

- Bottom with 16 levels peels to 12, 8, 4: its core is strange.
- Bottom with 17 levels peels to 13, 9, 5, 1: its core is down.
- A side of charm (a tetrahedron with 20 levels) peels to 16, 12, 8, 4. Its interior is exactly the bottom pattern with 16 levels: 816 points, checked by direct enumeration.
- A side of top passes through 20, 16, 12, 8, 4: charm's side, bottom, and strange.

3. The pairing of the generations

An up-type pattern with n levels contains, as its layers, the down-type patterns with up to n levels (section 2.2). Taking for each down-type quark the smallest up-type pattern that holds it:

Down-type	Level	Held by	Smallest
down	1	up (3), charm (20), top (72)	up
strange	4	charm, top	charm
bottom, read as 34 levels	34	top	top
bottom, read as 16 or 17 levels	16 or 17	charm, top	charm

Down pairs with up and strange with charm, as observed. Bottom pairs with top when its level is read as 34, three octaves above its measured frequency (section 8): 34 is above charm's 20, so only top holds it. Read as 16 or 17 levels, bottom is a layer of charm, and the smallest-stack rule would pair it with charm.

4. The three mixing angles

The amplitude for a pattern to be found in the configuration of a smaller pattern it contains is the square root of the fraction of its portions that the smaller one holds. The three angles are the three corner overlaps of section 2.3:

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sin θ12 = sqrt(C_down / C_strange) = sqrt(1/20)        = 0.223607
sin θ23 = sqrt(C_up / C_charm)     = sqrt(15/8,855)    = 0.041158
sin θ13 = sqrt(C_up / C_top)       = sqrt(15/1,215,450) = 0.003513
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The first is the Cabibbo angle: the down quark is the corner point of the strange tetrahedron. The relation sin θ_C = sqrt(m_d/m_s) was found empirically from quark masses in the late 1960s (Gatto, Sartori and Tonin; Oakes). Here it is a statement about the shape, and it uses counts, not masses: 1/20 is exact.

In Wolfenstein form, λ = sin θ12 and A = sin θ23 / λ^2:

	From the shapes	PDG 2026 global fit	Difference
λ	0.22361	0.22517 ± 0.00068	−0.7%
A	0.8232	0.826 (+0.017, −0.015)	−0.4%

5. The CP phase

The phase is the angle at which two sides of a regular tetrahedron meet:

`δ = arccos(1/3) = 70.53°.`

The down-type patterns are tetrahedra. In the regular embedding of the simplex, where every side is alike, adjacent sides meet at this angle.

6. The full matrix

Standard parametrisation with the three angles of section 4 and the phase of section 5:

	down	strange	bottom
up	0.974673	0.223605	0.003513
charm	0.223464	0.973843	0.041157
top	0.008684	0.040384	0.999146

Against the PDG 2026 values:

Element	Shapes	PDG global fit	Difference	PDG direct average	Difference
|V_ud|	0.974673	0.97431 ± 0.00016	+0.04%	0.97367 ± 0.00032	+0.10%
|V_us|	0.223605	0.22517 ± 0.00068	−0.69%	0.22431 ± 0.00085	−0.31%
|V_ub|	0.003513	0.003763 (+88, −83)e-6	−6.6%	0.00389 ± 0.00016	−9.7%
|V_cd|	0.223464	0.22503 ± 0.00068	−0.70%	0.2247 ± 0.0022	−0.55%
|V_cs|	0.973843	0.97345 ± 0.00016	+0.04%	0.969 ± 0.005	+0.50%
|V_cb|	0.041157	0.04189 (+81, −69)e-5	−1.75%	0.0407 ± 0.0013	+1.12%
|V_td|	0.008684	0.00863 (+19, −17)e-5	+0.63%	0.0086 ± 0.0002	+0.98%
|V_ts|	0.040384	0.04117 (+79, −68)e-5	−1.91%	0.0415 ± 0.0009	−2.69%
|V_tb|	0.999146	0.999115 (+29, −34)e-6	+0.003%	1.002 ± 0.024	−0.28%

CP violation and the unitarity triangle:

	Shapes	PDG 2026
Jarlskog invariant J	2.97e-5	(3.16 +0.13 −0.11)e-5
α	87.7°	84.1° (+3.7, −3.0)
γ	70.5°	66.4° (+2.7, −2.8)
sin 2β	0.691	0.710 ± 0.011

Eight of the nine magnitudes agree within 2 percent. The diagonal agrees to 0.04 percent or better. |V_td|, the element that depends most on the phase, agrees to 0.6 percent.

7. A second route to the phase

The up-to-strange transition has two routes through the shapes: through the down corner of strange, amplitude sqrt(1/20), and through the up corner of charm, amplitude sqrt(15/8,855). Writing

`|V_us| = | sqrt(C_d/C_s) − e^(iφ) sqrt(C_u/C_c) |`

and using the measured |V_us| = 0.22431 ± 0.00085 gives φ = 85.7° ± 1.2°, the unitarity angle α as measured, 84.1° (+3.7, −3.0).

8. The bottom quark

The bottom pattern is fixed by the geometry from three sides.

- It is the interior of one side of charm, exactly, with 16 levels (section 2.4).
- Its measured mass sits midway between 16 and 17 levels: with the down-family unit 4.67 MeV the mass needs a count of 895, between 816 and 969. The even blend gives 892.5, a mass of 4,168 MeV against 4,180 ± 30 MeV.
- The blend is also a single level read three octaves down. The identity S(2n + 2, 3) = 4 [S(n, 3) + S(n + 1, 3)] gives S(34, 3)/8 = (816 + 969)/2 = 892.5. A ring of 7,140 portions with a phase ripple of about two even steps puts its strongest component at harmonic 895.
- The cores of the two adjacent bottom levels are strange (16 levels) and down (17 levels).

The bottom pattern's own count does not enter the mixing matrix. The matrix needs only down, strange, up, charm and top, the corner overlaps, and the tetrahedral angle.

9. What the agreement says about the patterns

A count fixes a number of portions. The mixing matrix uses more than numbers: it uses corners, sides, interiors and the angle between sides. Those belong to the lattice points of a simplex. The matrix agrees with measurement because the portions inside a quark are laid out as a simplex, regular, with n levels in d directions, the down-type quarks as tetrahedra and the up-type quarks as their four-dimensional analogues.

The same counts already carry the masses, the charges, the colour structure and the generation law of IDWT. The quark mixing matrix is now among them.

10. Open items

- |V_ub| comes out 6.6 percent below the global fit (2.9 standard deviations) and 9.7 percent below the direct average (2.4 standard deviations).
- The level of the bottom pattern: the mass and the interiors tie it to 16 and 17, and the pairing with top needs a level above 20, which the three-octave reading at 34 supplies. Which description is the physical one is not settled here.
- Why the up corner of charm sets the 2–3 angle (section 4) and also enters the second route for the 1–2 transition (section 7).
- The lepton mixing angles do not follow from the lepton counts by the corner rule.