A Common Dimensional Structure for Particle Masses, Spin, Color and Charge

A Case for Investigating IDWT Sector Depths

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Abstract

A particle-mass prescription based on cumulative simplex counts assigns the observed families to spatial depths $2, 3, 4, 5, 6$ and 10. An apparently different calculation gives exactly the same set: within ten spatial dimensions, a single minimal real rotation spinor admits a nondegenerate, rotation-invariant first-order clock precisely at those depths. Two further complete-channel calculations single out depths three and four, the quark assignments, through exceptional three-component responses. A spatial spinor profile realizes spin one-half and has finite bound-direction norm above depth two. A six-normal-direction exterior construction supplies relative charge weights $0, -1, 2/3, -1/3$, with color singlets and two triplets of the same type; a separate depth-five construction connects neutrality with a handed kinetic sign.

We present these results together because they constrain different aspects of the same proposed dimensional organization. The mass formula, its electron-referenced scale prescription, numerical inputs, geometric calculations and compatibility conditions are given explicitly. Their shared premises can reduce the number of unrelated choices, although the results are not statistically independent. The charge weights do not yet uniquely assign every particle to a depth, and no common interacting action has been shown to select all occupied counts and reproduce all properties. The contribution is a reproducible convergence of conditional mathematical structures, together with concrete bridges needed to turn it into a physical theory. It is not a claim of experimental confirmation or a completed derivation of the Standard Model spectrum.

1 The convergence that motivates this paper

Why should the dimensions suggested by a particle-mass pattern also be dimensions distinguished by spinor clocks and color-like response geometry? This is the central question of the present paper. The useful observation is not merely that one can attach a geometrical picture to a list of masses. It is that several explicit calculations, with their own stated hypotheses, meet on the same small set of spatial depths.

The mass prescription is

$$m_{n,d} = \mu_d S(n,d), \qquad S(n,d) = \binom{n+d-1}{d}, \qquad n \geq 1. \tag{1}$$

Here d is the number of ordinary spatial directions involved in the bound pattern, n labels a completed level, and μ_d is its mass unit. The subscript on μ_d records the family assignment; it

does not require that the fundamental be a function of depth alone. In particular, the electron, muon and tau share one fundamental in this prescription despite occupying different depths.

The resulting assignment uses

$$\mathcal{D}_{\text{mass}} = \{2, 3, 4, 5, 6, 10\}. \tag{2}$$

A rotation-invariant first-order clock on one minimal real spinor gives

$$\mathcal{D}_{\text{clock}} = \{2, 3, 4, 5, 6, 10\}, \qquad 1 \le d \le 10. \tag{3}$$

No measured mass enters the clock calculation. This makes the agreement mathematically different from another fit to the same masses. It does not make the historical choice of a clock criterion a blind statistical test.

Depth d	Assigned family	Clock skew dimension	Middle-form channel rank	Lowest response channel rank
2	W, Z, H; photon boundary	1	1	0
3	down-type quarks	3	3	3
4	up-type quarks	3	3	3
5	neutrino mass states	3	10	5
6	electron, muon	1	10	15
10	tau	1	126	45

Table 1: The shared dimensional pattern. The two channel columns are different constructions, defined in Section 7; their three-component cases coincide with the quark depths. These ranks are not, by themselves, a derivation of an independent color force. The bottom mass remains an exception to the ordinary integer sequence.

The additional links are equally important to state accurately. A complete spatial spinor profile transforms with spin one-half, rather than only having a spinor label at its center. Its norm converges for $d > 2$, which includes all assigned matter depths. A shared exterior carrier produces the familiar relative charge values together with singlet and triplet multiplicities. This charge construction is compatible with the assignments but does not select all of them on its own. At depth five, a distinct angular construction relates neutrality and handedness. These results justify testing a common physical realization; they do not yet supply it.

1.1 Origin and contribution

According to the author's research account, the starting exercise asked an earlier GPT system to treat Standard Model mass-frequencies as audio data and search for harmonics. The spatial geometry and its depths were not specified in that prompt. Subsequent work organized the multipliers as cumulative simplex counts and examined their physical interpretation. This account explains how the question arose; it is not a recoverable, preregistered discovery protocol or a probability estimate.

The contribution here is the explicit comparison of the mass assignment with the later clock, spin, color and charge calculations, and an audit of their common premises. Clifford modules, Hodge decomposition, exterior algebra and critical Dirac profiles are established mathematical tools [5, 6, 7]. Their use here does not claim invention of those tools. The project-specific question is whether their consequences can be realized together by the field dynamics underlying Eq. (1).

Throughout, an *exact result* means a mathematical statement under specified hypotheses. A *physical premise* identifies a mathematical object with some part of a particle. A *numerical comparison* evaluates a prescribed formula against archived inputs. An *open bridge* is an

identification or dynamical result still needed. These categories make it possible to give the convergence its full weight without promoting a conditional theorem into an observation.

2 Physical setting and notation

Infinite-Dimensional Wave Theory (IDWT) is investigated here in ten ordinary spatial dimensions plus time, with a single field Ψ_∞. All physical constructions below belong to this one spacetime. The familiar three-dimensional observer view is a selected spatial view of that setting. A depth describes the dimensional involvement of a particle pattern, not a separate containing universe. Nested spatial subspaces are allowed; compactified dimensions and Kaluza–Klein towers are not part of this construction.

Complex components are convenient notation for pairs of real field components or oriented real planes. A representation space is algebraic bookkeeping for the field on spacetime, not an additional physical manifold. In particular, $SU(3)$ below names transformations of component labels; it does not name the topology of physical space. Interactions must ultimately arise from the same field. No mediator particles or particle-exchange mechanism are assumed.

The position of a particle, its internal field pattern, its expansion coefficients and a detector's time-averaged registration are different objects. The electron's external motion remains a literal deterministic trajectory in its six ordinary spatial dimensions, with the existing orbit-plus-precession picture as baseline. A classical spinor profile in this paper is not a probabilistic electron cloud or a replacement for that trajectory. Squared norms here measure classical component magnitude; they do not assert a Born position-probability law.

Symbol	Meaning
d	Bound spatial depth of a pattern.
$n,\ k = n-1$	Completed-level index and polynomial cutoff degree.
$C = S(n,d)$	Cumulative count, proposed as the number of participating portions.
$\mu_d,\ \omega_{0,d}$	Fundamental in mass and angular-frequency units.
j	Exterior degree in the charge construction; unrelated to n or k.
ℓ	Spatial angular degree; unrelated to the completed-level label.
$\mathcal{N}$	Integrated classical norm, not an integer count.
Σ_d	A specified spinor representation; its real or complex type is stated.

Table 2: Labels that must remain distinct. In particular, neither a spinor rank nor an exterior degree can silently replace the portion count.

Mass and frequency are related by the usual unit conversion

$$f_m = \frac{mc^2}{h}, \qquad \omega_m = \frac{mc^2}{\hbar}, \qquad \omega_{0,d} = \frac{\mu_d c^2}{\hbar}. \tag{4}$$

This conversion is not an IDWT discovery. The physical hypothesis concerns what field pattern selects and sustains that frequency, and how its complete mechanical response agrees with the assigned mass.

3 The count, the mass scales and the numerical spectrum

3.1 What the binomial coefficient counts

The number of monomials of total degree r in d variables is $\binom{r+d-1}{d-1}$. Counting all degrees through $k = n - 1$ gives

$$S(n,d) = \sum_{r=0}^{n-1} \binom{r+d-1}{d-1} = \binom{n+d-1}{d} = \binom{k+d}{d}. \tag{5}$$

Equivalently, this counts nonnegative integer tuples with $a_1 + \cdots + a_d \le k$. Adding a slack coordinate $a_0 = k - \sum_i a_i$ counts lattice points in a filled simplex. The slack coordinate is a counting device, not another physical dimension. The hockey-stick identity explains the cumulative count, while Pascal's identity gives its recursion. Neither identity chooses the occupied n.

The exact spacing is

$$\frac{S(n+1,d)}{S(n,d)} = \frac{n+d}{n}, \qquad \frac{S(n+1,d) - S(n,d)}{S(n,d)} = \frac{d}{n}. \tag{6}$$

For example, at the muon's $(n,d) = (35,6)$, the next rung is 17.14% higher. Thus a small error near that rung cannot be removed by an arbitrarily small change of its integer label.

3.2 A count can multiply an event rate without multiple resonances

Suppose C portions of one pattern each cycle at the same fundamental $1/T_0$, and each contributes one registered event per cycle. Counting all events, including simultaneous events with their multiplicity, gives

$$\lim_{T\to\infty} \frac{N_{\text{events}}(T)}{T} = \frac{C}{T_0}. \tag{7}$$

Fixed phase offsets need not be evenly spaced. A detector that merges coincident events or weights them unequally need not implement this additive readout; that is part of the physical bridge.

For the special evenly spaced case, a centered sawtooth $s(x) = \{x\} - 1/2$, with a fixed convention at the resets, obeys

$$\sum_{r=0}^{C-1} s\left(x + \frac{r}{C}\right) = s(Cx). \tag{8}$$

The identity follows by summing the integer parts. It realizes a waveform with C times the repetition rate from copies of one fundamental. Equally spaced sinusoidal copies instead cancel for $C > 1$; their linear sum is not this mechanism. Fourier harmonics of the sawtooth are a description of its shape, not additional independently resonating objects.

The proposed mass-frequency law is $\omega_m = C\omega_0$. What remains to be derived is why the physical population is a completed count (5), why its common clock persists, and why the actual interaction reads it as in Eq. (7). Additive counting alone does not derive mechanical inertia or a hard frequency cutoff.

3.3 The electron-referenced scale prescription

The maintained non-neutrino prescription uses the following dimensionless *sector coefficients*:

$$g_{22} = \frac{1445}{2}, \quad g_{33} = 8\sqrt{7}, \quad g_{44} = \frac{12}{\sqrt{7}}, \quad g_{66} = g_{10,10} = \frac{1}{4}. \tag{9}$$

These are proposed numerical coefficients; this calculation does not establish them as physical coupling constants. With reference counts

$$C_2^* = C_3^* = 1, \quad C_4^* = 15, \quad C_6^* = 18564, \tag{10}$$

the rule is

$$\mu_d = \frac{m_e}{C_d^*}\sqrt{\frac{g_{dd}}{g_{66}}} \quad (d = 2, 3, 4, 6), \qquad \mu_{10} = \mu_6. \tag{11}$$

The last equality is an additional common-kind premise: the charged leptons have the same fundamental. Equality of two g values by itself would not establish equality of scales with different reference counts.

Explicitly,

$$\begin{aligned}
\mu_2 &= m_e\sqrt{2890} = 27.47064959 \text{ MeV}/c^2, \\
\mu_3 &= m_e\sqrt{32\sqrt{7}} = 4.701857978 \text{ MeV}/c^2, \\
\mu_4 &= \frac{m_e}{15}\sqrt{\frac{48}{\sqrt{7}}} = 0.1451024874 \text{ MeV}/c^2, \\
\mu_6 &= \mu_{10} = \frac{m_e}{18564} = 27.52633861 \text{ eV}/c^2.
\end{aligned} \tag{12}$$

The only measured mass used to evaluate this chain is $m_e = 0.51099895000$ MeV/c^2. The W, down and up masses are comparison targets, not extra anchors. However, these coefficient choices were developed with the mass spectrum known, in an attempt to account for empirically fitted scales. Electron-only evaluation does not make their historical construction a blind prediction or a first-principles derivation.

Particle	d	n	C	Formula	Benchmark	Error (%)
W	2	76	2926	80379.1207	80369.2	+0.01234
Z	2	81	3321	91230.0273	91188	+0.04609
H	2	95	4560	125266.162	125200	+0.05285
d	3	1	1	4.70185798	4.70	+0.03953
s	3	4	20	94.0371596	93.5	+0.57450
u	4	3	15	2.17653731	2.16	+0.76562
c	4	20	8855	1284.88253	1273	+0.93343
t	4	72	1215450	176364.818	172570	+2.19900
e	6	13	18564	0.51099895	0.51099895000	+0.00000
μ	6	35	3838380	105.656548	105.6583755	-0.00173
τ	10	23	64512240	1775.78576	1776.93	-0.06439

Table 3: Formula values and archived comparison inputs, in MeV/c^2. The signed error is $100(m_{\text{formula}}/m_{\text{benchmark}} - 1)$. The electron is the sole measured anchor in this table.

Table 3 uses the archived benchmark inputs of the research account, rather than silently replacing them with a new world average. Quark mass conventions matter: light and heavy quark entries are scheme-dependent mass parameters, not direct measurements of free isolated quark rest energies. The Particle Data Group supplies the relevant empirical context and definitions [4]. The small printed residuals are not all within measurement uncertainties, and no experimental error bars have been folded into the table. In particular, the approximately 2.20% top discrepancy and the highly precise muon comparison must remain visible. For mechanism construction, the exact target is the formula itself; these residuals are not additional quantities being fitted here.

3.4 Neutrinos and the unresolved bottom

The three neutrino entries label mass states, not flavor eigenstates:

$$(n_1, n_2, n_3) = (10, 15, 22), \quad d = 5, \quad (C_1, C_2, C_3) = (2002, 11628, 65780). \tag{13}$$

The absolute scale cancels from the squared-splitting ratio,

$$R_\nu = \frac{C_2^2 - C_1^2}{C_3^2 - C_1^2} = 0.0303498422349. \tag{14}$$

An archived comparison uses $\Delta m_{21}^2 = 7.53 \times 10^{-5}$ and $\Delta m_{32}^2 = 2.455 \times 10^{-3}$ in eV^2/c^4; their corresponding ratio is 0.0297593170770. Keeping that distinction between 32 and 31 prevents a false comparison.

For a separate absolute-scale illustration, calibrating instead to $\Delta m_{21}^2 = 7.49 \times 10^{-5}\ \mathrm{eV}^2/c^4$ gives

$$\mu_5 = \sqrt{\frac{7.49 \times 10^{-5}}{11628^2 - 2002^2}}\ \mathrm{eV}/c^2, \quad (m_1, m_2, m_3) \simeq (1.513, 8.786, 49.701)\ \mathrm{meV}/c^2. \tag{15}$$

This calibration is independent of the electron-based non-neutrino chain. It must not be counted as a prediction of the neutrino absolute scale.

The ordinary down-type sequence has no comparably good bottom entry. The nearest logarithmic rung is $n = 17$, $C = 969$, about 8.92% above the archived 4183 MeV/c^2 benchmark. The expression $\mu_3\sqrt{S(16,3)S(17,3)} = \mu_3\sqrt{816 \cdot 969}$ lies nearer, but no physical mechanism selecting that geometric mean has been established. It is not used as an ordinary integer success. The extension $S(0,2) = 0$ can label zero photon rest mass; it does not assert zero photon energy or derive the photon dynamics.

4 How rigid is the assignment?

At a fixed depth, a common μ_d fixes all within-family ratios:

$$\frac{m_i}{m_j} = \frac{S(n_i, d)}{S(n_j, d)}. \tag{16}$$

For the charged leptons at depth six,

$$\frac{S(35,6)}{S(13,6)} = \frac{3838380}{18564} = \frac{3515}{17}. \tag{17}$$

A scale cannot improve one sibling independently of the others. Holding the listed indices fixed and minimizing the variance of $\log[M_i/S(n_i, d)]$ over $1 \le d \le 15$ selects $2, 3, 4, 6$ for the boson, down/strange, up/charm/top and electron/muon groups. This diagnostic fits a scale only for comparison; it is not the scale recipe used in Table 3.

A stronger, but still explicitly conditional, assignment argument allows every positive integer n. Let $a_d = \mu_d/m_e$. The fourth powers are exactly rational:

$$a_2^4 = 2890^2, \quad a_3^4 = 7168, \quad a_4^4 = \frac{256}{39375}, \quad a_6^4 = a_{10}^4 = \frac{1}{18564^4}. \tag{18}$$

Move whole family blocks bijectively among depths $2, 3, 4, 6, 10$, retain the scale law, require an exact electron reference, and retain the common charged-lepton fundamental. Exact electron matching forces depth six: at ten, the target count 18564 lies strictly between $S(7, 10) = 8008$

and $S(8,10) = 19448$; at the other depths the rational fourth-power brackets exclude equality. The distinct-depth and common-kind rules then place the tau at ten.

At depth three, the W target lies between rungs 45 and 46; at depth four it lies between 58 and 59. Both pairs exclude a 0.5% match. The boson block is therefore forced to two under this tolerance. The first depth-three rung already exceeds twice the up benchmark, forcing the up-type block to four and the remaining down-type block to three. Monotonicity and the exact fourth powers certify the bracketing without an arbitrary finite index cutoff. This demonstrates assignment rigidity under the specified rules; it does not make the bottom fit successful or prove uniqueness among every possible scale law and family partition.

The historical joint-accuracy calculation also tests absolute placement and within-family ratios together. Its conditional coverage is summarized in Appendix B. Because the coefficients, residual thresholds and search choices were developed after seeing the data, that coverage is not a global discovery probability. The geometric results below are not multiplied into it as independent odds.

5 An explicit spatial pattern with spin one-half

Let Hermitian Clifford matrices satisfy $\{\gamma_a, \gamma_b\} = 2\delta_{ab}\mathbf{1}$, and write $\mathcal{D} = -i\gamma_a\partial_a$ on $\mathbb{R}^d$. For $A > 0$, $a > 0$, and a constant unit spinor χ, consider

$$\psi_\chi(x) = A\left(1 + \frac{r^2}{a^2}\right)^{-d/2}\left(\mathbf{1} + i\frac{\gamma\cdot x}{a}\right)\chi, \qquad r = |x|. \tag{19}$$

This is a spatially varying classical field, of the critical Dirac-bubble form studied in established spin geometry [6]. Its scalar density and differential equation follow directly from the Clifford relation:

$$\begin{aligned}\rho = \psi^\dagger\psi &= A^2(1 + r^2/a^2)^{1-d},\\ \mathcal{D}\psi &= \frac{d}{a}(1 + r^2/a^2)^{-1}\psi = g\rho^{1/(d-1)}\psi, \qquad gA^{2/(d-1)} = d/a.\end{aligned} \tag{20}$$

For $d > 1$ this is the Euler equation of

$$\mathcal{F}[\psi] = \int_{\mathbb{R}^d}\left[\mathrm{Re}(\psi^\dagger\mathcal{D}\psi) - \frac{g(d-1)}{d}\rho^{d/(d-1)}\right]\mathrm{d}^d x. \tag{21}$$

The amplitude is constrained relative to the scale for a specified g; it is not fitted to a measured particle mass.

5.1 The rotation acts on the whole profile

For a spatial rotation R and its spin lift $U(R)$,

$$U(R)\psi_\chi(R^{-1}x) = \psi_{U(R)\chi}(x). \tag{22}$$

Thus the family of complete profiles carries the same rotation representation as the seed. In any observer three-plane, the generators $J_i = -i\gamma_j\gamma_k/2$ for cyclic (i,j,k) satisfy

$$\sum_{i=1}^{3} J_i^2 = \frac{3}{4}\mathbf{1}, \qquad U(2\pi) = -\mathbf{1}. \tag{23}$$

The full pattern therefore realizes the spin-one-half transformation law. The spinor premise already contains the half-angle action; this construction shows its compatibility with a nonconstant spatial field rather than deriving spinors from an unspecified scalar theory. It does not by itself fix an absolute classical angular momentum of $\hbar/2$ or establish fermionic statistics.

5.2 A dimension-dependent localization threshold

Radial integration, with $u = r^2/a^2$, gives

$$\mathcal{N} = \int_{\mathbb{R}^d} \rho \, \mathrm{d}^d x = A^2 a^d \pi^{d/2} \frac{\Gamma(d/2-1)}{\Gamma(d-1)}, \qquad d > 2. \tag{24}$$

The radial integrand behaves as r^{1-d} at infinity; convergence therefore requires $d > 2$. At two the integral to radius R is $\pi A^2 a^2 \log(1 + R^2/a^2)$. The finite coefficients at the assigned matter depths $3, 4, 5, 6, 10$ are respectively

$$\pi^2, \quad \frac{\pi^2}{2}, \quad \frac{\pi^3}{12}, \quad \frac{\pi^3}{24}, \quad \frac{\pi^5}{6720}. \tag{25}$$

This is a threshold for this profile family, not a universal theorem excluding all possible depth-two spinors. It is also a bound-direction norm: uniform extension through unused directions would not produce a finite full ten-space integral. The continuous scale a, the dimension-dependent nonlinear exponent, and the absence of selected n remain open common-action issues.

5.3 Integer-spin and polarization comparisons

Quadratic spinor components in a two-component observer representation decompose into a scalar and a three-vector, with rotation Casimirs zero and two. This supplies spin-zero and spin-one channels, not a complete identification of the Higgs, W and Z fields.

For a selected photon transverse plane E and normal direction K, a field polarization $\epsilon \in E_{\mathbb{C}}$ obeys $K \cdot \epsilon = 0$. Circular polarizations have rotation weights ± 1; their superpositions give linear or elliptical polarization. This is a physical-plane construction compatible with depth two. It is not the full massless-vector representation in ten spatial dimensions. The field equations must still select and sustain that plane, its envelope and its source-to-target connection; no independently traveling mediator is introduced here.

6 Which depths admit a minimal first-order clock?

A first-order clock needs more than a rotating picture: it needs a nondegenerate alternating form supplying the time-derivative term. Let V be one minimal irreducible *real* $\mathrm{Spin}(d)$ module with its positive invariant inner product. Seek a real matrix J such that

$$J^T = -J, \qquad \det J \neq 0, \qquad [J, U(R)] = 0 \quad \text{for every rotation } R. \tag{26}$$

Real Schur theory gives the commuting division algebra $\mathbb{R}$, $\mathbb{C}$ or $\mathbb{H}$. In these three cases its skew-adjoint subspace has dimension zero, one or three. In the latter two, a normalized nonzero skew element has $J^2 = -\mathbf{1}$. The standard real Clifford-module classification [5], reproduced by the finite matrix method in Appendix A, gives Table 4.

For real component coordinates q, the Lagrangian

$$L = \frac{1}{2} q^T J \dot{q} - H(q) \quad \Longrightarrow \quad J\dot{q} = \nabla H \tag{27}$$

has an invertible first-order kinetic structure exactly when such J exists. If $H = h(I)$ with $I = q^T q/2$, then

$$\dot{q} = -h'(I) J q, \qquad \frac{\mathrm{d}I}{\mathrm{d}t} = 0. \tag{28}$$

This is uniform rotation in the component plane at fixed norm. It establishes a clock structure, not a selected value of $h'(I)$.

d	Minimal real dimension	Commutant	Skew dimension
1	1	$\mathbb{R}$	0
2	2	$\mathbb{C}$	1
3	4	$\mathbb{H}$	3
4	4	$\mathbb{H}$	3
5	8	$\mathbb{H}$	3
6	8	$\mathbb{C}$	1
7	8	$\mathbb{R}$	0
8	8	$\mathbb{R}$	0
9	16	$\mathbb{R}$	0
10	32	$\mathbb{C}$	1

Table 4: Minimal real rotation modules. At four and eight, choose one chirality. The skew column, rather than the component count alone, determines whether the stated first-order clock exists.

Proposition 6.1. *Within $1 \le d \le 10$, one minimal irreducible real rotation spinor supports a nondegenerate rotation-invariant first-order clock exactly for $d = 2, 3, 4, 5, 6, 10$, if no additional canonical partner is supplied.*

This is the exact conditional agreement stated in Eq. (3). The assumptions do real work. Second-order dynamics, or repeated real-type modules paired into a skew form, need not obey this restriction.

6.1 Adding the full cumulative spatial band

The simplex count can be included without merely counting constant components. On

$$W = \Sigma_d \otimes P_{\le k}(\mathbb{R}^d), \qquad \dim_{\mathbb{R}} W = s_d S(n,d), \quad k = n-1, \tag{29}$$

use polynomials multiplied by a rotationally invariant decaying envelope. Here s_d is the minimal real spinor rank. At allowed depths, $J \otimes \mathbf{1}$ works throughout the complete band. At seven, eight and nine, the highest rotation representation is real type and appears once. Every invariant skew operator therefore retains a kernel.

More explicitly, put $r_\ell = s_d\binom{\ell+d-2}{d-2}$. At seven and nine the multiplicity is $m_\ell = k - \ell + 1$. At eight the two multiplicities are $m_\ell^+ = \lfloor (k-\ell)/2 \rfloor + 1$ and $m_\ell^- = \lfloor (k-\ell+1)/2 \rfloor$. The minimum kernel is the sum of r_ℓ over odd multiplicities, and satisfies

$$\frac{K_{\min}}{s_d S(n,d)} \ge \frac{d(d-1)}{(n+d-1)(n+d-2)} > 0. \tag{30}$$

Thus a complete finite band on that minimal fibre does not remove the exclusion. It also does not select a particular finite cutoff.

6.2 The remaining isolation problem

A minimal real active spinor must be distinguished from a full massive Clifford field. The real 32-component Spin(10) half-spin module restricts to lower depths with repeated copies; its skew commutant dimensions at seven, eight and nine are $6, 2, 1$. Those multiplicities restore clocks. The full massive field used later has 32 *complex* components, or 64 real components, and is different again.

Consequently the clock theorem becomes a physical depth selector only if the dynamics isolates the required minimal active pattern. Keeping this question visible strengthens the proposed research test: the same minimal-branch issue also appears in the next color calculation.

7 Two geometric routes to the quark depths

7.1 Complete middle forms and the exceptional triple

A differential m-form encodes oriented m-dimensional elements. In an oriented Euclidean $2m$-space, the Hodge star maps each middle form to its oriented complementary form and obeys $*^2 = (-1)^m \mathbf{1}$. Write the space as an odd $(2m-1)$-plane V plus unit normal n. Every middle form is $A + B \wedge n^\flat$, with $A \in \Lambda^m V$ and $B \in \Lambda^{m-1} V$. For $\lambda^2 = (-1)^m$,

$$E_\lambda(B) = *_V B + \lambda^{-1} B \wedge n^\flat \tag{31}$$

is a star eigenform, with norm squared $2\|B\|^2$. It follows that the complete chiral channel and its odd-dimensional shadow have rank

$$r_m = \binom{2m-1}{m-1} = \frac{1}{2}\binom{2m}{m}. \tag{32}$$

For odd depth the use of a containing even carrier is an additional premise, not an automatic consequence of odd-dimensional space.

The ranks are $1, 3, 10, 35, 126, \ldots$, and $r_{m+1}/r_m = 4 - 2/(m+1) > 1$. Therefore the complete channel has rank three only for $m = 2$, corresponding to depths three and four. This is an all-positive-depth counting result, not a scan with an unexplained upper cutoff. On the assigned depths, its ranks are the fourth column of Table 1.

A related instantaneous contact filter is

$$H_* = \frac{1}{2}(* + *^\dagger) = \begin{cases} *, & m \text{ even}, \\ 0, & m \text{ odd}. \end{cases} \tag{33}$$

Among the assigned depths it retains three/four and cancels two, five/six and ten. Seven/eight would also have an active rank-35 channel, so this filter is useful in combination with the clock premise, rather than a replacement for it.

7.2 The lowest nontrivial spinor response

There is a second uniform calculation. On traceless endomorphisms of a minimal complex rotation spinor, using one chirality in even dimensions, define

$$\mathcal{C}_d(A) = \sum_{a<b} [S_{ab}, [S_{ab}, A]], \qquad S_{ab} = -\frac{i}{4}[\gamma_a, \gamma_b]. \tag{34}$$

For a Clifford blade of grade t, precisely $t(d-t)$ bivectors have one index in the blade and one outside it. Each contributes one copy, giving $\mathcal{C}_d(A_t) = t(d-t)A_t$. Even blades give the odd-dimensional endomorphisms; at even depth a chiral branch identifies complementary grades and halves an even middle grade.

At odd $d \geq 3$ the lowest channel has rank d; at even $d \geq 6$ it has rank $\binom{d}{2}$. Depth four has the exceptional half-middle rank three. Again, exactly three and four supply a complete triple. Using full paired Dirac carriers instead changes the displayed ranks to $2, 12, 8, 20, 12, 20$. Thus the result genuinely depends on the minimal branch, as does the clock selector.

d	Complex spinor rank	Lowest positive eigenvalue	Channel rank
2	1	none	0
3	2	2	3
4	2	4	3
5	4	4	5
6	4	8	15
10	16	16	45

Table 5: The lowest nontrivial response of a minimal rotation spinor. The physical premise is occupation of this complete lowest channel.

7.3 From a spatial triple to independent color

A three-component spatial response has an $so(3)$ or $su(2)$ rotation algebra, not automatically the eight-generator algebra $su(3)$. Independent color requires three equivalent complex component labels with norm- and volume-preserving transport. That transport must be realized by the same physical field rather than inferred merely from the number three.

One compatible source of such labels uses six ordinary normal directions to a spatial four-base. Pairing them as three complex directions gives the Clifford decomposition

$$\Sigma_{10} = \Sigma_4 \otimes \Lambda^*\mathbb{C}^3, \qquad \dim_{\mathbb{C}} \Sigma_{10} = 4 \cdot 8 = 32. \tag{35}$$

Complete normal degree two is a conjugate-fundamental triplet. Its transformations commute with observer spin. For its projector P,

$$P\gamma_\mu P = \gamma_\mu^{(4)} \otimes \mathbf{1}_3 \quad (1 \le \mu \le 4), \qquad P\gamma_i P = 0 \quad (5 \le i \le 10). \tag{36}$$

The second identity occurs because a normal Clifford vector changes normal parity. A fixed triplet therefore retains tangent kinetics at depths three/four but cannot replace the complete ten-direction field. At depth two the same restriction is kinetically possible; a source selection rule, such as Eq. (33), is still needed.

With $\mathrm{Tr}(T_A T_B) = \delta_{AB}/2$, the exact color identity is

$$\sum_A (T_A)_{ij}(T_A)_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{jk} - \frac{1}{3}\delta_{ij}\delta_{kl}\right). \tag{37}$$

It gives $C_F = 4/3$, $C_A = 3$, and

$$\sum_A T_A \otimes T_A = \frac{1}{2}(\mathsf{S} - \mathbf{1}/3), \tag{38}$$

where S swaps the two factors. The antisymmetric/symmetric eigenvalues are $-2/3, 1/3$; a triplet–conjugate pair has singlet/octet values $-4/3, 1/6$. Three triplets have the invariant epsilon contraction. Both quark families must use the same triplet type for those determinant singlets. These statements establish color algebra; neither the sign of an algebraic eigenvalue nor a singlet contraction alone derives attraction or confinement.

7.4 Explicit compatibility with a clock and a local source

On the real form of $\mathbb{C}^2 \otimes \mathbb{C}^3$, commuting with both observer spin and independent color leaves the complex commutant. Define

$$J_0 = i\mathbf{1}, \quad J_1 = (\epsilon \otimes \mathbf{1}_3)\mathcal{K}, \quad J_2 = J_0 J_1, \quad \epsilon = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{39}$$

where $\mathcal{K}$ is complex conjugation. For $J(a) = a_0 J_0 + a_1 J_1 + a_2 J_2$ with $\sum a_i^2 = 1$ and $R_A = i\mathbf{1}_2 \otimes T_A$, direct contraction yields

$$\frac{1}{12} \sum_A \|[J(a), R_A]\|^2_{\mathrm{HS,real}} = \frac{10}{3}(a_1^2 + a_2^2). \tag{40}$$

Independent color removes the extra quaternionic clock directions but preserves $J = \pm i\mathbf{1}$. This is a concrete example of shared assumptions restricting one another without making the constructions incompatible.

A local same-field source can also implement the Hodge filter while retaining full kinetic components. For an even carrier set

$$\begin{aligned} \Gamma_{2m} &= i^m \gamma_1 \cdots \gamma_{2m}, \quad P_\chi = (\mathbf{1} + \chi\Gamma_{2m})/2, \\ K_I &= i^{m(m-1)/2} \gamma_{i_1} \cdots \gamma_{i_m}, \quad B_{A,I} = \tfrac{1}{2}\{K_I, T_A\}, \\ J_{\chi,A,I} &= \Psi^\dagger P_\chi B_{A,I} P_\chi \Psi. \end{aligned} \tag{41}$$

The spatial form index I and color label A remain distinct. The proposed polynomial contact is

$$V \operatorname{vol}_{2m} = \gamma \sum_{A,\chi} \chi J_{\chi,A} \wedge J_{\chi,A}, \qquad \gamma > 0. \tag{42}$$

For odd m each self-wedge vanishes. For even m, the chiral Clifford identity gives $*J_{\chi,A} = \chi J_{\chi,A}$, hence $V = \gamma \sum_{A,\chi} \|J_{\chi,A}\|^2 \geq 0$. At three/four this equals $\gamma \sum_A \|H_* J_A\|^2$. The projectors act only in the source; the kinetic field stays full. The odd shadow uses contraction with the parent normal before restriction. This is a constructive compatibility witness under a chosen contact law, not a proof that every other allowed interaction vanishes.

8 A common carrier for the relative charges

8.1 The charge values and color types together

Let W represent three paired ordinary normal planes, so $\dim_{\mathbb{R}} W = 6$, and let $D = \det W$. Consider the exterior carrier

$$F = \mathbf{1} \oplus \Lambda^2 W, \qquad E = F \otimes (\mathbf{1} \oplus D^{-1}). \tag{43}$$

Under the central rotation $W \mapsto e^{i\theta} W$, a two-form acquires weight two and D^{-1} acquires weight minus three. Dividing all weights by three, so that the determinant branch is one charge unit, gives Table 6.

Block	Weight	Relative charge	Color type
$\mathbf{1}$	0	0	singlet
$\Lambda^2 W$	2	2/3	$\overline{\mathbf{3}}$
D^{-1}	−3	−1	singlet
$\Lambda^2 W \otimes D^{-1}$	−1	−1/3	$\overline{\mathbf{3}}$

Table 6: The relative charge spectrum from one exterior construction. Multiplicities are $1, 3, 1, 3$. A common observer spin doublet doubles them without changing the charges. The choice of triplets versus conjugate triplets is conventional, but their equality of type is essential.

Both triplets have the same $SU(3)$ type because the determinant character is trivial on $SU(3)$. The weighted trace vanishes: $0 + 3(2/3) - 1 + 3(-1/3) = 0$. The construction produces

the relative values and color multiplicities together. It is compatible with neutral depth-five states, charged singlets at six and ten, and the two quark families at three/four. It does not, by itself, force each block to occupy its assigned depth.

There are two further distinctions. First, the branch doublet in Eq. (43) is not the observer spin doublet and does not supply a derived weak interaction. Second, a component eigenvalue does not fix a classical integrated charge:

$$Q_{\text{integrated}} = \int \Psi^\dagger Q \Psi \, \mathrm{d}^{10}x \tag{44}$$

still scales with the field norm. Equal observed electron, muon and tau charges require a common physical source law and normalization, even though their relative component weight can be the same.

8.2 Why six normal directions and a four-base are plausible

A conditional normal-frame test asks for a same-chirality alternating vector bilinear

$$\beta : \Lambda^2 S \longrightarrow N_{\mathbb{C}}, \qquad \dim_{\mathbb{R}} N = 2m, \tag{45}$$

which is nonzero for every independent pair of spinors. The same-chirality vector bilinear is alternating precisely for $m \equiv 3 \pmod 4$. For fixed nonzero s, universal nonvanishing requires $\ker \beta(s, \cdot) = \mathbb{C}s$, giving

$$2^{m-1} - 1 \leq 2m. \tag{46}$$

Among the map-bearing dimensions, only $m = 3$ satisfies this bound; $\Lambda^2\mathbb{C}^4 \simeq \mathbb{C}^6$ verifies sufficiency. The universal-pair premise matters: selecting special pairs permits higher dimensions.

There is also an explicit four-plus-six spatial split related to the pure-spinor geometry studied by Krasnov [7]. A pure spinor has a maximal isotropic Clifford annihilator. In an exterior chart, an orthogonal pair whose *every* linear combination is pure can be written $u = 1$, $v = e_{12}$. Indeed, expanding $1 + tv = \exp B(t)$ forces v to be a decomposable two-form with $v \wedge v = 0$. For a normalized pure spinor, form the spatial skew matrix $(J_\psi)_{ij} = -i\psi^\dagger \gamma_i \gamma_j \psi$ off the diagonal. For this pair, J_u, J_v commute and

$$P_B = \frac{\mathbf{1} + J_u J_v}{2}, \quad \operatorname{rank}_{\mathbb{R}} P_B = 4, \qquad P_N = \frac{\mathbf{1} - J_u J_v}{2}, \quad \operatorname{rank}_{\mathbb{R}} P_N = 6. \tag{47}$$

These projectors act on actual spatial directions. No extra physical space is introduced, and no gauge-particle interpretation of the external mathematics is imported.

The normal six-plane is *not* the electron's bound six-plane. For example, a base $\langle e_1, \ldots, e_4 \rangle$ has normal complement $\langle e_5, \ldots, e_{10} \rangle$, whereas a nested bound six-plane could be $\langle e_1, \ldots, e_6 \rangle$. The latter needs an additional normal complex line. The pure pair alone does not select the complete nested flag of particle depths. Moreover, normal $SU(3)$ rotations preserving that fixed bound six-plane reduce to $U(2)$; preserving a fixed bound five-plane reduces them to $SU(2)$. Independent color transport therefore cannot simply be identified with rotations preserving every fixed particle subspace.

8.3 Keeping all ten spatial derivatives

A pointwise component label can fail to commute with spatial kinetics. An exact free-field repair includes the spatial angular derivatives. Set $\hbar = c = 1$ and use the full 32-complex-component field with

$$H_D = -i\gamma_i \partial_i + M\beta, \quad \{\beta, \gamma_i\} = 0, \quad \beta^2 = \mathbf{1}, \tag{48}$$

where $i = 1, \ldots, 10$. Write $L_{ab} = -i(x_a\partial_b - x_b\partial_a)$ and $J_{ab} = L_{ab} + S_{ab}$. On the four-base define

$$\Gamma_4 = \gamma_1\gamma_2\gamma_3\gamma_4, \quad X_4 = \gamma_a x_a, \quad D_4 = \gamma_a\partial_a, \quad E_4 = x_a\partial_a,$$
$$B_4 = 2\sum_{a<b\le 4} S_{ab}L_{ab} + \frac{3}{2} = \frac{3}{2} - X_4D_4 + E_4, \qquad K_4 = \Gamma_4 B_4,$$
$$Q_{\text{ang}} = \frac{1}{3}(K_4 + N_6), \qquad N_6 = J_{56} + J_{78} + J_{9,10}. \tag{49}$$

This Γ_4 is the negative of the earlier i^m chirality convention at $m = 2$; it relabels the two base branches. Clifford differentiation gives $\{B_4, D_4\} = 0$, hence

$$[Q_{\text{ang}}, H_D] = 0. \tag{50}$$

The operator also commutes with total base rotations and the appropriate total normal $U(3)$. In the lowest base angular channel, with zero normal orbital degree and even exterior degree $j = 0, 2$,

$$q = \frac{\pm 3/2 + j - 3/2}{3}. \tag{51}$$

This reproduces $-1, -1/3, 0, 2/3$ with multiplicities $2, 6, 2, 6$ and observer spin one-half. The mass parameter M is supplied here; this free operator is not the missing mass-selection operator.

The repair has precise limits. Higher base angular degree shifts the charges; the lowest degree is an extra selection. The operator is angular about a chosen origin and is not generally translation invariant. Nor does free conservation guarantee interacting conservation. A reviewed example with $V = \lambda\rho^2/2$, three orthogonal positive-β Gaussian components of width L, centers $(b, 0), (0, b), (-b, -b)$ in the base plane, and polarization signs $(1, -1, 1)$ has zero centroid and ordinary total torque but

$$\frac{\mathrm{d}\langle Q_{\text{ang}}\rangle}{\mathrm{d}t} = \frac{2\lambda b^2 A^4}{3L^2}\left(\frac{\pi L^2}{2}\right)^5 \left[e^{-b^2/L^2} - e^{-5b^2/(2L^2)}\right] > 0 \tag{52}$$

for positive λ, b. Recentring on the centroid does not fix it. This excludes that interaction as a conserving realization of this charge, not every possible same-field interaction. A physical electric source remains a substantive open problem.

9 Depth five: a link between neutrality and handedness

Depth five has a distinctive relation to an observer three-plane: its remaining two directions form one rotation plane. Among assigned matter depths it is the only case in which the entire nontrivial normal rotation algebra $so(d-3)$ is abelian. At larger depths, choosing just one such plane needs additional structure.

Using three sets of Pauli matrices as component notation, choose

$$\alpha_i = \tau_3 \otimes \kappa_3 \otimes \sigma_i \quad (i = 1, 2, 3),$$
$$\alpha_4 = \tau_3 \otimes \kappa_1 \otimes \mathbf{1}, \qquad \alpha_5 = \tau_3 \otimes \kappa_2 \otimes \mathbf{1}. \tag{53}$$

Define $\eta = \tau_3 \otimes \mathbf{1} \otimes \mathbf{1}$, $K = -i\alpha_4\alpha_5$, and the handed kinetic sign $C_5 = -i\alpha_1\alpha_2\alpha_3$. Direct multiplication gives

$$C_5 = \eta K, \qquad Q_5^{(0)} = \frac{\eta + K}{3}, \qquad (Q_5^{(0)})^2 = \frac{2}{9}(\mathbf{1} + C_5) = \frac{4}{9}P_R. \tag{54}$$

Here C_5 is a sign operator, unrelated to the portion count C. Opposite η and K signs give both neutrality and the left-handed kinetic sign. Restoring spatial rotation in the normal plane (u, v) gives

$$\mathcal{Q}_5 = \frac{\eta + 2J_\perp}{3}, \qquad J_\perp = -i(u\partial_v - v\partial_u) + K/2. \tag{55}$$

Both terms commute with the free five-space Dirac operator and observer spin. This is a different candidate from Q_{ang}; their common physical source identification has not been derived.

For a specified rotational normal texture with angular connection

$$\mathcal{A}_\theta = \frac{w(r/a)^{2w}}{1+(r/a)^{2w}}, \qquad \text{flux} = 2\pi w, \qquad \lambda_\eta = -\eta, \tag{56}$$

the normal zero modes are

$$f_m \propto \frac{r^m e^{-i\eta m\theta}}{\sqrt{1+(r/a)^{2w}}}, \quad 0 \le m < w-1, \quad K = -\eta, \quad \mathcal{Q}_5 f_m = -\frac{2\eta m}{3} f_m. \tag{57}$$

At infinity, the radial norm behaves as $\int r^{2m-2w+1}\mathrm{d}r$; this proves the upper bound. The smallest integer winding that supports a normalizable mode is $w = 2$. Only $m = 0$ then remains, so it is neutral with the left-handed kinetic sign. More generally, exactly one neutral angular mode survives for $1 < |\lambda| w \le 2$; larger effective flux permits charged angular modes. The connection here is a conditional same-field texture to realize, not an independently postulated mediator field.

The five-dimensional profile of Eq. (19) with a $C_5 = -1$ seed is also annihilated by $\mathcal{Q}_5$: changes of normal spin are compensated by the spatial angular term. Its two handed component norm fractions are 3/4 and 1/4, not measured helicity probabilities. The bubble and the texture are not already a common solution; adding the connection to the bubble equation generally leaves a nonzero residual. Nor is the proposed generator unique: $\eta/3$ alone also commutes with the free kinetics and does not give the same neutrality. Thus this is a testable link between two properties under explicit assumptions, not a completed neutrino or weak-interaction derivation.

10 Other properties and the remaining dynamical bridge

10.1 Opposite-charge patterns and absolute spin

The free ten-space equation admits a concrete same-equation partner map. In the Clifford convention of Appendix A, choose $\beta = i\gamma_1 \cdots \gamma_{10}$ and put

$$B_e = \gamma_2\gamma_4\gamma_6\gamma_8\gamma_{10}, \qquad A = (\beta\gamma_4)B_e, \tag{58}$$

and let R_4 reflect x_4. Then $\Psi^{\#}(t,x) = A\Psi^*(-t, R_4 x)$ preserves the free equation, energy and norm while reversing the angular charge. It survives the stated real scalar-density nonlinearities of the source account [1]. This supplies a candidate conjugate construction; identifying its full physical behavior with observed antiparticles remains open.

For a radial field $\Psi = F(r)\chi + iG(r)\gamma_r\chi$ in D spatial dimensions, the integrated angular momentum includes a spatial part. Writing $\mathcal{N}_F = \int |F|^2$, $\mathcal{N}_G = \int |G|^2$ and $\mathcal{N} = \mathcal{N}_F + \mathcal{N}_G$, the decomposition is

$$\begin{aligned} J_{ab} &= \mathcal{N} S^{\mathrm{seed}}_{ab}, \\ J^{\mathrm{spin}}_{ab} &= \left[\mathcal{N}_F + \frac{D-4}{D}\mathcal{N}_G\right] S^{\mathrm{seed}}_{ab}, \\ L_{ab} &= \frac{4}{D}\mathcal{N}_G S^{\mathrm{seed}}_{ab}. \end{aligned} \tag{59}$$

At ten, the G part divides as 3/5 component spin and 2/5 spatial angular momentum. This reinforces the need to treat the full pattern, while also showing why the absolute spin needs a physical normalization. Conditional magnetic-moment diagnostics exist, but no calculation here derives the observed magnetic moments from the complete IDWT source law.

10.2 Localization does not yet select the mass spectrum

A useful later construction shows how explicit spatial support can be added without treating a particle as a coefficient vector alone. In ten dimensions let

$$s_+ = \max\left(1 - \frac{|x - X|^2}{R^2}, 0\right), \quad F_R = \sqrt{p_*}\, s_+^4, \quad p_* = \frac{\Gamma(14)}{\pi^5 R^{10}\Gamma(9)}. \tag{60}$$

Then $\int F_R^2 \mathrm{d}^{10}x = 1$. For the proposed shape energy

$$E_{\mathrm{sh}}[f] = \gamma \int \left(|\nabla f|^2 + A f^{3/2} - B f^{7/4}\right) \mathrm{d}^{10}x, \quad A = \frac{64 p_*^{1/4}}{R^2}, \quad B = \frac{1024 p_*^{1/8}}{7R^2}, \tag{61}$$

the profile has energy $104\gamma/(7R^2)$. A source-account finite nonlinear bound holds for exact-norm, fixed-support perturbations $f = F_R(1 + h)$ with $|h| \le 0.01$, after imposing its weighted boundary and translation-slice conditions:

$$E_{\mathrm{sh}}[f] - E_{\mathrm{sh}}[F_R] \ge \frac{7\gamma}{R^2} \int \frac{(f - F_R)^2}{s} \mathrm{d}^{10}x. \tag{62}$$

The integral is over the support where $s > 0$. This is a restricted energy basin, not a proof of global time-dependent stability. Every portion in this comparison has ten-dimensional support; the lower particle depths and integer population are not derived from it.

A proposed collective Hamiltonian makes the missing normalization particularly clear:

$$H_C = \sqrt{c^2\Pi^2 + B_C^2}, \qquad B_C = \Omega\,[P + \hbar(C - 1)] + U, \qquad P = \sum_a I_a. \tag{63}$$

On the supplied leaf $P = \hbar$, $U = 0$, this gives rest mass $C\hbar\Omega/c^2$. The desired count dependence is thus realized conditionally, but its offset $\hbar(C - 1)$ has been supplied as a constitutive premise. The count, action leaf and clock-preserving conditions on U still require dynamics. Equation (63) is a proposal, not a derived mass operator. These later dynamical ideas do not enter the depth-convergence claim or its finite verification.

For a complete localized, stationary-rest-frame field with conserved stress and vanishing boundary terms, the integrated spatial stresses vanish. Together with an appropriate covariant dynamics, this provides the usual route from rest energy to inertial response. Applying that route to a chosen formula does not derive the formula's occupied counts. The required operator must select the patterns and their frequencies, not merely encode the known spectrum in its coefficients.

11 Do the shared assumptions fit together?

The most useful assessment separates demonstrated coexistence from assumptions that have not yet been realized by one action.

Table 7: Dependency and compatibility audit.

Shared choice	Consequences already calculated	Remaining physical bridge
Minimal active spinor	Clock set matches the mass depths; lowest response triples occur at three/four.	Isolate the branch dynamically within the full spatial field; unrestricted multiplicities change both tests.
One common fundamental per kind	Count law fixes family ratios; electron, muon and tau share a mass unit.	Sustain common-frequency portions and select $C = S(n, d)$ without inserting the occupied labels.
Four-base plus six normal directions	Independent normal triplets, relative charges and observer spin can coexist algebraically.	Select the whole nested bound-depth flag and its compatible transport.
Independent color with observer spin	Same-type triplets and determinant singlets; a complex first-order clock survives.	Derive the interaction and its strength, reciprocal response and confinement behavior.
Selected chiral source contact	Hodge filter acts at the assigned quark depths while full kinetic components remain.	Establish this contact from a common law rather than assume it is the only permitted one.
Lowest angular charge sector	Correct charge values with spin doublets and full free spatial kinetics.	Interacting conservation, translation behavior, lowest-sector selection and integrated charge normalization.
Depth-five normal plane	A conditional generator links neutrality with a handed sign and a minimal normalizable texture mode.	Derive the generator, texture and actual neutrino response in the common field.

There are positive compatibility results. Color and a clock can coexist, as Eq. (40) verifies. Charge and observer spin commute on the displayed lowest blocks. The source-only projection preserves the full kinetic field while implementing a depth-sensitive contact. The exterior carrier gives both required triplets the same type. These are stronger statements than finding four separate attractive analogies.

There are also unresolved joins. The minimal real clock module cannot silently be equated with the full complex massive field. The normal six-plane cannot silently be equated with the electron's bound six-plane. A complete spatial spinor can carry spin one-half without fixing the norm-dependent absolute angular momentum. Relative charges can be exact without an interacting electric conservation law. These distinctions define what a successful common theory must preserve.

Reusing a justified choice for several consequences is explanatory economy: fewer unrelated decisions would be needed. Reusing a premise also creates dependence. The clock and lowest-channel results share minimality; charge and color share an exterior structure. Their probabilities cannot be multiplied as though each resulted from an independent experiment. The question is instead whether a small explicit set of choices yields several correct consequences *simultaneously*.

12 Research tests and conclusion

The proposed structure can be advanced or weakened by definite tests:

1. Derive minimal active-pattern isolation under a common ten-space kinetic law. If unavoidable repeated modules restore the excluded clocks, the present clock criterion will not serve as a physical selector.

2. Obtain completed counts and occupied indices from a sustained same-field solution. Reproducing a binomial dimension is insufficient unless its physical participation and frequency are established.

3. Derive the sector coefficients or replace them with a better common scale law, while preserving the existing family-ratio constraints. A new choice must face all targets together, including the bottom exception.

4. Construct an interacting, reciprocal charge source with the required spin and color compatibility. It must address the explicit failure in Eq. (52) and explain equal charged-lepton source strengths despite different counts.

5. Test persistence, translation and response of actual particle patterns. Long-lived electron candidates and unstable particles need different lifetime behavior; universal eternal stability is not required.

6. Freeze the resulting model before testing new quantitative consequences. Better agreement obtained by additional fitted choices must remain distinguishable from an out-of-sample result.

The central finding is a convergence of constrained structures. The depth set inferred within the mass prescription coincides with the minimal-spinor first-order clock set. The quark depths coincide with two exceptional three-component constructions. Spatial spin-one-half profiles, charge/color carriers and a depth-five neutrality construction fit substantial parts of the same picture, with explicit qualifications. This warrants investigation of a common dynamical origin. It does not yet demonstrate that these spatial patterns are realized in nature.

The proposed case is therefore specific: the dimensional assignments organize more than masses, and the additional mathematics is sufficiently explicit to reproduce, challenge and extend. The decisive next result would be a common field law that selects the patterns while retaining their demonstrated algebraic compatibility.

A Reproduction: numerical inputs and finite matrix checks

The numerical masses in Table 3 are printed inputs, not values fetched during a calculation. Exact integers and coefficients are given in Eqs. (5)–(11). Standard SI conversion uses $c = 299792458\ \mathrm{m\,s^{-1}}$, $h = 6.62607015 \times 10^{-34}\ \mathrm{J\,s}$ and $1\ \mathrm{eV} = 1.602176634 \times 10^{-19}\ \mathrm{J}$.

The accompanying Python program `particle-geometry-checks.py` uses Python 3, NumPy and SymPy. Running it writes `particle-geometry-checks.json`. It checks count identities, formula masses, fixed-index depth diagnostics, exact all-positive-index brackets, neutrino ratios, rational sawtooth identities, minimal clock commutants, complete-band kernels, Hodge ranks, lowest adjoint ranks, color contractions, the clock/color compatibility defect and the lowest charge/spin blocks. It does not solve a nonlinear particle equation or validate the physical premises. The accompanying paper-table program regenerates the printed numerical table from that report.

For direct independent implementation, let X, Y, Z be the Pauli matrices. The ten Clifford

matrices are

$$\gamma_{2q+1} = Z^{\otimes q} \otimes X \otimes \mathbf{1}^{\otimes(4-q)},$$
$$\gamma_{2q+2} = Z^{\otimes q} \otimes Y \otimes \mathbf{1}^{\otimes(4-q)}, \qquad q = 0, \ldots, 4. \tag{64}$$

For the three normal planes use

$$n_a = \frac{\mathbf{1} - i\gamma_{2a+3}\gamma_{2a+4}}{2}, \quad a = 1, 2, 3. \tag{65}$$

On even total degree $j = 0, 2$, diagonalize

$$Q_0 = \frac{(3/2)\Gamma_4 + \sum_a n_a - 3/2}{3}. \tag{66}$$

It has eigenvalues $-1, -1/3, 0, 2/3$ with multiplicities $2, 6, 2, 6$. The three observer generators $-i\gamma_2\gamma_3/2$, $-i\gamma_3\gamma_1/2$, $-i\gamma_1\gamma_2/2$ commute with Q_0 and have total Casimir $3/4$. The differential repair follows from $D_4X_4 + X_4D_4 = 2E_4 + 4$ and $[E_4, D_4] = -D_4$.

The real clock table can be reproduced without numerical nullspaces. Use tensor words in $\mathbf{1}, X, Z, A$, where $A = \left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)$. Label these matrices $0, 1, 2, 3$. Two words commute exactly when the number of slots containing different nonidentity entries is even; a word is skew exactly when it contains an odd number of A factors. Seven negative-square, anticommuting real matrices are the words

$$300,\ 230,\ 223,\ 213,\ 103,\ 131,\ 132. \tag{67}$$

Pair products of the first five, six or seven give the corresponding spin generators. For a chiral eight-module use the seven words and their pair products. For nine, prepend A to the seven and add $X \otimes \mathbf{1}_8, Z \otimes \mathbf{1}_8$; for ten prepend Z to those nine and add $X \otimes \mathbf{1}_{16}$. Pair products give the rotation generators. At three/four use the quaternionic words $30, 23, 13$; at two use A. Enumerating commuting words and their skew parity gives Table 4 exactly. At one the minimal module is real rank one.

For the Hodge test, enumerate all m-subsets of $\{1, \ldots, 2m\}$. Map a subset to its complement with the orientation sign of the concatenated permutation. This signed permutation matrix obeys $*^2 = (-1)^m\mathbf{1}$ and $*^T = (-1)^m*$. For color, use the six off-diagonal matrices $(E_{ij} + E_{ji})/2$, $(-iE_{ij} + iE_{ji})/2$ and the two diagonal matrices $\mathrm{diag}(1, -1, 0)/2$, $\mathrm{diag}(1, 1, -2)/(2\sqrt{3})$. These recipes reproduce the color and joint-clock contractions without requiring any inaccessible project data.

B Scope of the retrospective joint-accuracy calculation

The existing rigid test asks how much of a *declared* random-spectrum space meets the observed absolute and ratio accuracies under the fixed electron-referenced model. It does not ask for the probability that IDWT is true or that the historical research process would find some pattern of comparable apparent quality.

The electron is fixed. Ten other masses are sampled as logarithmic ratios $x_i = \log(M_i/m_e)$ independently uniform on $[0, \log R]$, with ordering within the WZH, down/strange, up/charm/top and muon/tau groups. Their ordered joint density is $3!2!3!2!/(\log R)^{10} = 144/(\log R)^{10}$. The bottom and the separately calibrated neutrinos are not successful scored entries in this ten-target calculation.

For the actual formula values P_i^*, set

$$r_i = \log(P_i^*/M_i), \quad \delta_i = |r_i|, \quad \eta_{ij} = |r_j - r_i|. \tag{68}$$

A candidate complete assignment must simultaneously satisfy the individual absolute-log-error bounds δ_i and the declared within-family difference bounds η_{ij}. Sector scales are not refitted. The same one assignment must satisfy the whole conjunction; dependent ratio events are not multiplied separately. The search retains the six remaining whole-family permutations and all relevant rung tuples, including repeated and nonnearest choices.

The preserved union estimator covers this acceptance region by N_{box} equal-volume component boxes of volume $V = 1.0409457956707493 \times 10^{-29}$. With K the number of boxes containing a sampled point, it uses

$$p = \frac{144 N_{\mathrm{box}} V}{(\log R)^{10}} \mathbb{E}\left[\frac{\mathbf{1}_{\text{ordered window}}}{K}\right]. \tag{69}$$

The inverse multiplicity corrects overlaps. The historical calculation used one million draws for each window:

R	Sufficient rung bound	Estimated coverage	Sampling interval
10^6	194	2.837×10^{-20}	$[2.673, 2.983] \times 10^{-20}$
10^7	616	7.328×10^{-18}	$[6.709, 7.719] \times 10^{-18}$

These are reported results from the reviewed joint calculation in the public joint-accuracy account [3]; they were not rerun for this manuscript. The intervals are 99% simultaneous sampling bounds with the documented conditional numerical guard, not fully directed-rounding mathematical enclosures. Reproduction of that full union calculation requires its published source packet; the finite companion checker does not perform it.

The small coverage has a precise meaning: the fixed template with these joint observed-precision tolerances occupies little of this log-uniform ordered spectrum space. The dependence on R is substantial and shown explicitly. Choosing the coefficients, tolerances, model class and comparisons after observing the spectrum is not accounted for by these numbers. A calibrated global discovery probability would require that larger selection procedure. No numerical odds are assigned here to the combined mass/clock/color/charge convergence.

C Glossary and reading index

Bound depth, d (Sections 2, 3)
Number of ordinary spatial directions involved in the bound pattern; not a component rank or a compactified space.

Clock commutant (Section 6)
Matrices commuting with every rotation on the specified real spinor. Its skew elements supply the proposed first-order clock.

Clifford matrices (Sections 5, A)
Matrices whose anticommutators encode the Euclidean spatial metric. They express spinorial rotations and first-order spatial derivatives.

Color versus a spatial triple (Section 7)
A three-dimensional response channel is not automatically independent $SU(3)$ color. A separate compatible triplet action and transport are needed.

Complete channel (Section 7)
The whole representation selected by a stated criterion, rather than three favorable components chosen from a larger space.

Exterior degree, j (Section 8)
Number of factors in an antisymmetric exterior component. Its rotation weight supplies the charge arithmetic; it is not a particle index.

Fundamental and registered rate (Section 3)
Each portion cycles at the fundamental. Counting every portion's event gives C times that rate under the additive-registration premise.

Hodge star (Section 7)
Metric-and-orientation operation that associates a differential form with its complementary oriented form.

Minimal module (Section 6)
One smallest irreducible real rotation-spinor representation. Adding copies can change the clock and response conclusions.

Normal six-plane (Section 8)
Six ordinary directions perpendicular to a chosen four-base in ten-space. It is distinct from a particle's six bound directions.

Pure spinor (Section 8)
A spinor with a maximal isotropic Clifford annihilator. Here its bilinears help specify projectors onto actual spatial subspaces.

Relative charge weight (Section 8)
Eigenvalue of a proposed component or differential generator. A conserved, properly normalized physical electric source requires further dynamics.

Simplex count, $S(n, d)$ (Section 3)
The cumulative number of degree-at-most-$n-1$ monomials in d variables, equivalently the specified filled-simplex lattice count.

Data, provenance and computational assistance

This manuscript synthesizes the public IDWT particle-geometry case [1] and its corrected readable account [2], using the research state frozen on 1 November 2026 (catalog snapshot 3669). Exact source hashes and the executed checker are included in the editable companion. The earlier account and finite checker received separate internal mathematical review; this new manuscript is an authorial synthesis and has not itself received independent peer review. Later constitutive proposals are identified as such. Computational checks establish their stated finite identities, not the physical interpretation.

The research and preparation used AI assistance for mathematical exploration, calculation, checking and exposition. Scientific claims, assumptions and open problems are stated in the manuscript rather than being warranted by the systems that helped produce it.

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